Dr. Ela Sharma's

SAT Math

Manual and Workbook

For the New SAT

ISBN: 9798296315427

Published in the United States of America.

Version SE5.19

Volume discounts are available for schools, teachers, tutors, and learning agencies.
Email: tutorhubllc@gmail.com.

Additional resources and information in this book, including errata, are available at www.tutorhubllc.com.

Disclaimers:
The author of this book has made best effort to provide accurate and complete content and assumes no liability for errors and inaccuracies that may occur.

Answers

Section 1 – Variables and Expressions in Linear Equations

Category 1 – Solve Variables and Expressions in Linear Equations

1. D

Determine the equation:

$$8 + 14c = 96 \rightarrow 14c + 8 = 96$$

2. B

Simplify the expression within parentheses:

$$y + 1 = (-3 \times y) - (-3 \times 3) \rightarrow y + 1 = -3y + 9$$

Isolate the variables on one side and solve:

$$y + 3y = 9 - 1 \rightarrow 4y = 8 \rightarrow y = 2$$

3. D

Isolate the variables on one side and solve: for ease, add the variables and the integers on the left-side first.

$$-b - 2 = b \rightarrow -2 = b + b \rightarrow 2b = -2 \rightarrow b = -1$$

4. C

Cross multiply and solve: For ease, add the variables on the left-side first.

$$\frac{6a}{5} = 2 \rightarrow 6a = 2 \times 5 \rightarrow 6a = 10 \rightarrow a = \frac{10}{6} = \frac{5}{3}$$

5. B

Determine the equation and solve:

$$11 + a = 5 \times 9 \rightarrow 11 + a = 45 \rightarrow a = 45 - 11 = 34$$

6. C

Isolate the variables on one side and solve:

$$\frac{x}{2} - \frac{3x}{10} = \frac{2}{5} \rightarrow \frac{5x}{10} - \frac{3x}{10} = \frac{2}{5} \rightarrow \frac{2x}{10} = \frac{2}{5} \rightarrow x = 2$$

7. B

Substitute the value of y in the equation and solve:

$$3x(4) - 10x = 14 \rightarrow 12x - 10x = 14 \rightarrow 2x = 14 \rightarrow x = 7$$

8. D

Cross multiply:

$$3x \times 7 = 4 \times y \rightarrow 3x \times 7 = 4y$$

Isolate the variables on one side and solve: Note that if $3x$ is moved to the right-side, the answer can be obtained.

$$\frac{4y}{3x} = 7$$

9. A

Simplify the left-side fraction and isolate $\frac{y}{x}$ on one side:

$$\frac{\left(\frac{x}{7}\right)}{3} = y \rightarrow \frac{x}{7} \div 3 = y \rightarrow \frac{x}{7} \times \frac{1}{3} = y \rightarrow \frac{x}{21} = y \rightarrow \frac{y}{x} = \frac{1}{21}$$

10. C

Isolate the variables on one side and solve:

$$x = 11 - 8 = 3$$

Evaluate which answer is the same: Answer choice C when simplified is $x = 3$.

11. C

Isolate the variables on one side and solve:

$$15m - 9m = 24 \rightarrow 6m = 24 \rightarrow 3m = 12$$

12. A

Simplify the expression within parentheses:

$$(3 \times 2x) - (3 \times 8) = (2 \times 2x) - (2 \times 7) \rightarrow 6x - 24 = 4x - 14$$

Isolate the variables on one side and solve:

$$6x - 4x = -14 + 24 \rightarrow 2x = 10 \rightarrow x = 5$$

13. C

Isolate identical expressions on one side and solve:

$$4(n + 5) - 3(n + 5) = 38 \rightarrow (n + 5) = 38$$
$$\big((n + 5) = 38\big) \times 3 \rightarrow 3(n + 5) = 114$$

14. B

Isolate the variables on one side and solve:

$$\frac{2x}{3} - \frac{x}{5} = \frac{7}{5} \rightarrow \frac{10x}{15} - \frac{3x}{15} = \frac{7}{5} \rightarrow \frac{7x}{15} = \frac{7}{5} \rightarrow \frac{x}{3} = 1 \rightarrow x = 1 \times 3 = 3$$

15. D

Isolate identical expressions on one side and solve:

$$\frac{5}{8}(t - 3) - \frac{3}{8}(t - 3) = 42 \rightarrow \frac{2}{8}(t - 3) = 42 \rightarrow \frac{1}{4}(t - 3) = 42 \rightarrow \left(\frac{1}{4}(t - 3) = 42\right)4 \rightarrow t - 3 = 168$$

16. 36

Determine the value of n: Plug in $a = 12$ and $b = m$.

$$a = \frac{b}{n} \rightarrow 12 = \frac{m}{n} \rightarrow 12n = m \rightarrow n = \frac{m}{12}$$

Determine a when $b = 3m$: plug in above value of n:

$$a = 3m \div \frac{m}{12} \rightarrow a = 3m \times \frac{12}{m} = 36$$

Section 2 – Lines and Linear Functions

Category 2 – Slope Intercept Form Equation of a Line

1. <u>D</u>

Determine the slope from the graph:

$$\frac{\text{rise}}{\text{run}} = \frac{1}{3}$$

This eliminates answer choices B and C that have negative slope.

Determine the y-intercept: from graph $= 1$.

This eliminates answer choice A. Note that equation D when divided by 3 is $y = \frac{1}{3}x + 1$.

2. <u>C</u>

Determine the slope of line p:

Slope of line $l = 2$ (from the given equation).

Slope of line $p = 3 \times 2 = 6$. This eliminates answer choices A and B.

Determine the y-intercept of line p: Since line p passes through the point $(0, 2)$, the y-intercept $= 2$. This eliminates answer choice D.

3. <u>C</u>

Determine the slope of the line: It is given slope $= 6$.

This eliminates answer choices A and B.

Determine the y-intercept of the line: Since $(0,-11) =$ y-intercept, $b = -11$. This eliminates answer choice A.

4. <u>B</u>

Determine the slope: Set up a slope equation using the points $(-1, 2)$ and $(1, 6)$.

$$\frac{6-2}{1-(-1)} = \frac{4}{2} = 2$$

This eliminates answer choices A and D.

Determine the y-intercept: Plug slope $= 2$ and any given point into $y = mx + b$ equation.

$$6 = (2 \times 1) + b \ \rightarrow \ b = 6 - 2 = 4$$

This eliminates answer choice C.

5. <u>B</u>

Determine the x-intercept: Set $y = 0$ and solve for x.

$$0 = \frac{2x+24}{4} - 3 \ \rightarrow \ 0 = \frac{1}{2}x + 6 - 3 \ \rightarrow$$

$$0 = \frac{1}{2}x + 3 \ \rightarrow \ -3 = \frac{1}{2}x \ \rightarrow \ x = -6$$

6. <u>B</u>

Since the line passes through origin y-intercept $= 0$. This eliminates answer choices C and D.

Using the points $(0, 0)$ and $(2, 6)$, the slope is 3. Hence, the equation is $y = 3x$. This is same as answer choice B.

7. <u>17/5 or 3.4</u>

Rearrange as $y = mx + b$:

$$y = \frac{42x}{5} + \frac{19}{5} - 5x = \frac{42}{5}x + \frac{19}{5} - \frac{25}{5}x = \frac{17}{5}x + \frac{19}{5}$$

Hence, slope $= \frac{17}{5}$ or 3.4.

8. <u>1.5 or 3/2</u>

Determine the x-intercept:

Let the x-intercept $= (x, 0)$. Set up 2 slope equations.

$$\frac{7-(-3.5)}{4.5-0} = \frac{0-(-3.5)}{x-0} \ \rightarrow \ \frac{10.5}{4.5} = \frac{3.5}{x} \ \rightarrow$$

$$10.5x = 3.5 \times 4.5 \ \rightarrow \ 10.5x = 15.75 \ \rightarrow \ x = 1.5$$

9. <u>4</u>

Determine the y-intercept: Plug the given point $(3, -5)$ and slope $= -3$ into $y = mx + b$ equation.

$$-5 = (-3 \times 3) + b \ \rightarrow \ -5 = -9 + b \ \rightarrow \ b = 4$$

10. <u>1</u>

Determine the slope:

-8 is the x-intercept. Hence the point is $(-8, 0)$.

Set up a slope equation using the points $(-8, 0)$ and $(3, 11)$.

$$\frac{11-0}{3-(-8)} = \frac{11}{11} = 1$$

11. <u>C</u>

Determine the slope: For every 3 units increase of x from left to right, y increases by 2 units up. Hence, slope is positive.

$$\frac{\text{rise}}{\text{run}} = \frac{2}{3}$$

This eliminates answer choices A and D.

Determine the y-intercept: Plug in the given point $(2, 5)$ and slope $= \frac{2}{3}$ in $y = mx + b$ equation.

$$5 = \left(\frac{2}{3} \times 2\right) + b \ \rightarrow \ 5 = \frac{4}{3} + b \ \rightarrow \ b = 5 - \frac{4}{3} \ \rightarrow \ \frac{11}{3}$$

This eliminates answer choice B.

12. <u>D</u>

Determine the slope: Divide the equation by a.

$$y = -2x + b$$

Since the slope is negative, the line on the graph will slant downward from left to right. This eliminates answer choices A and C.

Determine the y-intercept: Since $b > 1$, answer choice B can be eliminated.

Category 3 – Standard Form Equation of a Line

1. A

Determine the slope: Determine the slope from the graph:

$$\frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$$

Look for the answer choice where $-\frac{A}{B} = 2$.

This eliminates answer choices B, C, and D. In correct answer choice A, $-\frac{A}{B} = -\frac{6}{-3} = 2$.

2. B

Determine the y-intercept:

$$\frac{C}{B} = \frac{-5}{-2} = \frac{5}{2}$$

3. C

Determine the x-intercept: Set $y = 0$ and solve for x.

$$2x + (3 \times 0) = 4c \;\rightarrow\; 2x = 4c \;\rightarrow\; x = 2c$$

The coordinates are $(2c, 0)$.

4. D

Slope is given as $-\frac{3}{2}$.

Look for answer choice where $-\frac{A}{B} = -\frac{3}{2}$. This eliminates answer choices A and C.

Determine the y-intercept: Since the line intersects the x-axis at 4, $(4, 0)$ is a point on the line. Plug this point and slope $= -\frac{3}{2}$ in $y = mx + b$ equation.

$$0 = \left(-\frac{3}{2} \times 4\right) + b \;\rightarrow\; 0 = -6 + b \;\rightarrow\; b = 6$$

This eliminates answer choice B.

5. B

Solve for a: Plug the given point $(4, 1)$ into the equation.

$$4 - (a \times 1) = a \;\rightarrow\; 4 - a = a \;\rightarrow\; 2a = 4 \;\rightarrow\; a = 2$$

Category 4 – Points on a Line with Unknown Coordinates

1. A

Set up a slope equation and solve: Set up a slope equation using points $(2, 2p)$ and $(5, p - 1)$ and equate to $= -2$.

$$\frac{(p-1) - 2p}{5 - 2} = -2 \;\rightarrow\; \frac{-p - 1}{3} = -2 \;\rightarrow$$
$$-p - 1 = -2 \times 3 \;\rightarrow\; -p - 1 = -6 \;\rightarrow\; p = 5$$

Determine $p - 1$: $5 - 1 = 4$.

2. D

Set up slope equations and solve:

To calculate m, set up a slope equation using the points $(1, 4)$ and $(5, m)$ and equate to slope = 3.

$$\frac{m - 4}{5 - 1} = 3 \;\rightarrow\; \frac{m - 4}{4} = 3 \;\rightarrow\; m - 4 = 12 \;\rightarrow\; m = 16$$

To calculate n, set up a slope equation using the points $(1, 4)$ and $(n, 7)$ and equate to slope = 3.

$$\frac{7 - 4}{n - 1} = 3 \;\rightarrow\; \frac{3}{n - 1} = 3 \;\rightarrow\; 3(n - 1) = 3 \;\rightarrow$$
$$n - 1 = 1 \;\rightarrow\; n = 2$$

Determine $m + n$: $16 + 2 = 18$.

3. C

Set up slope equations and solve: Since the slope is not given, set up two slope equations and equate them. Set up one slope equation using the points $(1, 3)$ and $(4, a)$, and another using the points $(4, a)$ and $(7, a + b)$.

$$\frac{a - 3}{4 - 1} = \frac{(a + b) - a}{7 - 4} \;\rightarrow\; \frac{a - 3}{3} = \frac{b}{3} \;\rightarrow$$
$$a - 3 = b \;\rightarrow\; a - b = 3$$

4. B

Set up slope equations and solve: Set up one slope equation using the points $(1, 2)$ and $(5, 10)$ (this will give the slope), and another using the points $(1, 2)$ and $(s, 6)$. Equate the two slope equations.

$$\frac{6 - 2}{s - 1} = \frac{10 - 2}{5 - 1} \;\rightarrow\; \frac{4}{s - 1} = \frac{8}{4} \;\rightarrow\; \frac{4}{s - 1} = 2 \;\rightarrow$$
$$2(s - 1) = 4 \;\rightarrow\; s - 1 = 2 \;\rightarrow\; s = 3$$

5. D

Set up a slope equation and solve: Set up a slope equation using the points $(0, 0)$ and $(3, 3a)$ and equate to slope = 3.

$$\frac{3a - 0}{3 - 0} = 3 \;\rightarrow\; \frac{3a}{3} = 3 \;\rightarrow\; 3a = 9$$

Hence, $(3, 3a) = (3, 9)$.

6. 8

Set up a slope equation and solve: Set up a slope equation using the points $(0, 2)$ and $(x, -2)$ and equate to slope $= -\frac{1}{2}$.

$$\frac{-2 - 2}{x - 0} = -\frac{1}{2} \;\rightarrow\; \frac{-4}{x} = -\frac{1}{2} \;\rightarrow$$
$$(-1 \times x) = (-4 \times 2) \;\rightarrow\; -x = -8 \;\rightarrow\; x = 8$$

Category 5 – Slope of Parallel Lines

1. <u>B</u>

Determine the slope: Slope of line k is

$$\frac{\text{rise}}{\text{run}} = -\frac{1}{1} = -1$$

Since line l is parallel to line k, the slope of line $l = -1$. This eliminates answer choices C and D.

Determine the y-intercept of line l: Plug the given point $(6, -4)$ and slope $= -1$ into $y = mx + b$ equation.

$$6 = (-1 \times -4) + b \ \rightarrow\ 6 = 4 + b \ \rightarrow\ b = 2$$

This eliminates answer choice A.

2. <u>D</u>

Determine the slope: Slope of line a is

$$-\frac{A}{B} = -\frac{6}{3} = -2$$

Since line b is parallel to line a, the slope of line $b = -2$.

Look for the equation where $-\frac{A}{B} = -2$.

This eliminates answer choices A, B, and C.

3. <u>B</u>

Determine the slope: The slope of line $s = 4$. Since line t is parallel to line s, the slope of line $t = 4$.

Determine the y-intercept of line t: Plug the given point $(3, 5)$ and slope $= 4$ into $y = mx + b$ equation.

$$5 = (4 \times 3) + b \ \rightarrow\ 5 = 12 + b \ \rightarrow\ b = -7$$

The coordinates are $(0, -7)$.

4. <u>A</u>

Set up slope equations and solve: Since lines m and n are parallel lines, they have the same slope. Set up one slope equation using the points $(c, 3)$ and $(6, 5)$ on line m, and another using the points $(c, 1)$ and $(8, 4)$ on line n. Equate the two slope equations.

$$\frac{5-3}{6-c} = \frac{4-1}{8-c} \ \rightarrow\ \frac{2}{6-c} = \frac{3}{8-c} \ \rightarrow$$
$$2(8-c) = 3(6-c) \ \rightarrow\ 16 - 2c = 18 - 3c \ \rightarrow$$
$$c = 2$$

5. <u>C</u>

Determine the slope of line p:

$$-\frac{A}{B} = -\left(\frac{2a}{3} \div -\frac{b}{7}\right) = -\left(\frac{2a}{3} \times -\frac{7}{b}\right) = \frac{14a}{3b}$$

The slope of a parallel line will be the same.

Evaluate the slope in each answer choice to match above:

Answer choice A is not correct. See below.

$$-\frac{A}{B} = -\left(-\frac{4a}{3} \div -\frac{2b}{7}\right) = -\left(-\frac{4a}{3} \times -\frac{7}{2b}\right) = -\frac{14a}{3b}$$

Answer choice B is not correct. See below.

$$-\frac{A}{B} = -\left(\frac{2a}{7} \div -\frac{2b}{3}\right) = -\left(\frac{2a}{7} \times -\frac{3}{2b}\right) = \frac{3a}{7b}$$

Answer choice C is correct. See below.

$$-\frac{A}{B} = -\left(\frac{7a}{3} \div -\frac{b}{2}\right) = -\left(\frac{7a}{3} \times -\frac{2}{b}\right) = \frac{14a}{3b}$$

No need to check answer choice D.

Category 6 – Slope of Perpendicular Lines

1. <u>A</u>

Determine the slope: The slope of line $p = 3$. Since line q is perpendicular to line p, the slope of line $q = -\frac{1}{3}$. This eliminates answer choices B and D.

Determine the y-intercept of line q: Since line q passes through the point $(0, -4)$, the y-intercept $= -4$.

2. <u>B</u>

Determine the slope: Slope of line p is

$$\frac{\text{rise}}{\text{run}} = -\frac{1}{2}$$

Since line t is perpendicular to line p, the slope of line $t = 2$.

Determine the x-intercept: Let the x-intercept $= (x, 0)$. Set up a slope equation using the points $(x, 0)$ and $(1, 4)$ and equate to slope $= 2$.

$$\frac{4-0}{1-x} = 2 \ \rightarrow\ \frac{4}{1-x} = 2 \ \rightarrow\ 2(1-x) = 4 \ \rightarrow$$
$$1 - x = 2 \ \rightarrow\ x = -1$$

3. <u>C</u>

Determine the slope: Set up a slope equation using the points $(-1, 3)$ and $(3, 1)$ on line segment l.

$$\frac{1-3}{3-(-1)} = \frac{-2}{4} = -\frac{1}{2}$$

Since line h is perpendicular to line segment l, the slope of line $h = 2$. This eliminates answer choices A and B.

Determine the range of the y-intercept: The perpendicular line h can be any line passing between the two endpoints of line segment l.

Plug each end point and slope $= 2$ into $y = 2x + b$ equation.

For point $(-1, 3)$:

$$3 = (2 \times -1) + b \ \rightarrow\ 3 = -2 + b \ \rightarrow\ b = 5$$

For point $(3, 1)$:

$$1 = (2 \times 3) + b \ \rightarrow\ 1 = 6 + b \ \rightarrow\ b = -5$$

Hence, the y-intercept of line h can be any number between -5 and 5. This eliminates answer choice D.

4. D

Determine the slope: Write the equation of line s in the correct standard form before proceeding.

$$9y + 3x = 3 \rightarrow 3x + 9y = 3$$

Slope of line s is

$$-\frac{A}{B} = -\frac{3}{9} = -\frac{1}{3}$$

Since line t is perpendicular to line s, slope of line $t = 3$.

Determine the y-intercept of line t: Plug the given point $(-2, 5)$ and slope $= 3$ into $y = mx + b$ equation.

$$5 = (3 \times -2) + b \rightarrow 5 = -6 + b \rightarrow b = 11$$

Since $x = 0$ at y-intercept, the coordinates are $(0, 11)$.

5. C

Determine the slope: Set up a slope equation for line m using the points (3, 7) and (4, 9).

$$\frac{9-7}{4-3} = \frac{2}{1} = 2$$

Since line n is perpendicular to line m, the slope of line $n = -\frac{1}{2}$. This eliminates answer choices B and D.

Since line n passes through origin answer choice A can be eliminated. The equation in answer choice C when divided by 2 is $y = -\frac{1}{2}x$.

Category 7 – Linear Functions

1. B

Determine the slope: Set up a slope equation using the points $(-2, -5)$ and $(2, 3)$.

$$\frac{3-(-5)}{2-(-2)} = \frac{8}{4} = 2$$

This eliminates answer choices A and D.

Determine the y-intercept: Plug any given point and slope $= 2$ into $y = mx + b$ equation.

$$3 = (2 \times 2) + b \rightarrow 3 = 4 + b \rightarrow b = -1$$

This eliminates answer choice C.

2. C

Slope = relationship between number of miles and month.

3. A

Set up a slope equation and solve: Let the unknown point be $(-2, y)$.

Set up a slope equation using the points $(1, 4)$ and $(-2, y)$ and equate to slope $= 3$.

$$\frac{y-4}{-2-1} = 3 \rightarrow \frac{y-4}{-3} = 3 \rightarrow$$

$$y - 4 = -9 \rightarrow y = -5$$

4. B

Set up slope equations and solve: Let the unknown point be $(1, y)$.

Set up one slope equation using the points $(-2, 6)$ and $(3, -4)$, and another using the points $(3, -4)$ and $(1, y)$. Equate the two slope equations.

$$\frac{-4-6}{3-(-2)} = \frac{y-(-4)}{1-3} \rightarrow -2 = \frac{y+4}{-2} \rightarrow$$

$$(-2 \times -2) = y + 4 \rightarrow 4 = y + 4 \rightarrow y = 4 - 4 = 0$$

5. D

Determine a slope: Select any two points from the table and set up a slope equation. Below equation is set up using the points (2, 6) and (4, 9).

$$\frac{9-6}{4-2} = \frac{3}{2}$$

Determine the y-intercept: Plug any of the given point and slope $= \frac{3}{2}$ into $y = mx + b$ equation. Point $(2, 6)$ is used in the equation below.

$$6 = \left(\frac{3}{2} \times 2\right) + b \rightarrow 6 = 3 + b \rightarrow b = 3$$

Since $x = 0$ at y-intercept, the coordinates are $(0, 3)$.

Category 8 – Graph Transformations of Linear Functions

1. C

Determine the equation of f:

Determine the slope using any two given points.

$$\text{Slope} = \frac{149-114}{17-12} = \frac{35}{5} = 7$$

Determine the y-intercept of f:

$$y = mx + b \rightarrow 114 = (7 \times 12) + b \rightarrow b = 30$$

Equation is $y = 7x + 30$.

Determine the equation of g: 10 units down is

$$y = 7x + 30 - 10 \rightarrow y = 7x + 20$$

Determine the x-intercept of g: Plug in $y = 0$.

$$0 = 7x + 20 \rightarrow 7x = -20 \rightarrow x = -\frac{20}{7}$$

2. C

Left by 3 units, changes $2x$ to $2(x + 3)$.

Up by 2 units, changes -4 to $-4 + 2 = -2$.

Hence, translated function is $f(x) = 2(x + 3) - 2$.

3. A

Reflection across x-axis changes $y = 4x + 1$ to $-y = 4x + 1 \rightarrow y = -(4x + 1) \rightarrow y = -4x - 1$.

4. D

Reflection across y-axis changes $f(x) = x + 2$ to $f(x) = -x + 2$. This eliminates answer choices A and B with positive slope and answer choice C with y-intercept -2.

Section 2 – Drill

1. B

Determine the slope:

$$\frac{\text{rise}}{\text{run}} = -\frac{1}{2}$$

This eliminates answer choices C and D.

Determine the y-intercept:

$$y\text{-intercept from graph} = 2$$

This eliminates answer choice A.

2. A

Since the slope of line p is undefined, it is a vertical line. Only answer choice A is the equation of a vertical line.

3. 95

Set the given equation to 76:

$$\frac{4}{5}b = 76 \rightarrow b = \frac{76 \times 5}{4} = 95$$

4. D

The negative reciprocal of $-\frac{3}{10}$ is $\frac{10}{3}$.

5. B

Determine the slope:

The slope of parallel lines is the same. Hence,

$$\text{slope} = -3$$

This eliminates answer choices A and C.

Determine the y-intercept: Since the line passes through $(0, -2)$, the y-intercept $= -2$.

This eliminates answer choice D.

6. C

Determine the slope:

$$\frac{C}{B} = \frac{4}{-b} = -\frac{4}{b}$$

Determine b: Equate the y-intercept from the above equation with the given y-intercept $= -6$.

$$-\frac{4}{b} = -6 \rightarrow -6b = -4 \rightarrow b = \frac{4}{6} = \frac{2}{3}$$

7. D

Determine the slope:

$$\frac{\text{rise}}{\text{run}} = -\frac{4}{10} = -\frac{2}{5}$$

Look for the equation where $-\frac{A}{B} = -\frac{2}{5}$. This eliminates answer choices A, B, and C.

8. C

Set up slope equations and solve: Let the unknown point be $(-6, y)$.

Select any two points from the table to set up slope equations. Below slope equations are set up using the points $(-1, -4)$ and $(-3, 0)$, and the points $(-3, 0)$ and $(-6, y)$. Equate the two slope equations.

$$\frac{0-(-4)}{-3-(-1)} = \frac{y-0}{-6-(-3)} \rightarrow \frac{4}{-2} = \frac{y}{-3} \rightarrow$$

$$-2y = 4 \times -3 \rightarrow -2y = -12 \rightarrow y = 6$$

9. B

Determine the slope: From the equation of function g, slope $= 3$. Since the lines on the graphs of function f and g are parallel, they have the same slope $= 3$.

Set up a slope equation for function f and solve: Let the unknown point be $(1, y)$.

Set up a slope equation using the points $(3, 3)$ and $(1, y)$ and equate to slope $= 3$.

$$\frac{y-3}{1-3} = 3 \rightarrow \frac{y-3}{-2} = 3 \rightarrow y - 3 = -6 \rightarrow y = -3$$

10. B

Determine the x-intercept: Set $y = 0$ and solve.

$$2 \times 0 = 5x - 15 \rightarrow 5x = 15 \rightarrow x = 3$$

11. B

Determine the slope: Select any two points from the table and set up a slope equation. Below equation is set up using the points $(0, 2k)$ and $(3, 3k)$.

$$\frac{3k - 2k}{3 - 0} = \frac{k}{3}$$

This eliminates answer choices C and D.

Determine the y-intercept: Since $(0, 2k)$ is a point in the table, y-intercept $= 2k$. This eliminates answer choice A.

12. A

Determine the slope: The slope of line $k = 1$. Since line l is parallel to line k, the slope of line $l = 1$.

Look for the equation where $-\frac{A}{B} = 1$. This eliminates answer choices B, C, and D.

13. C

Determine the slope: Use any two points from the table.

$$\frac{84.95 - 44.95}{8 - 4} = \frac{40}{4} = 10$$

Determine the y-intercept: Plug slope $= 10$ and any point from the table into $y = mx + b$ equation.

$$44.95 = (10 \times 4) + b \rightarrow 44.95 = 40 + b \rightarrow b = 4.95$$

14. D

Determine the slope as rise/run:

$$\frac{\text{rise}}{\text{run}} = \frac{3}{2}$$

Evaluate each answer choice: Start with $(-4a, 5b)$ and determine the next point on the line. Continue till a point matches an answer choice. Since the points in the answer choices are greater than the given point, start with upward points.

$(-4a + \text{run}), (5b + \text{rise}) \rightarrow (-4a + 2), (5b + 3)$. This point is not an answer choice.

$(-4a + 2 + 2), (5b + 3 + 3) \rightarrow (-4a + 4), (5b + 6)$. This is answer choice D.

15. C

Determine the equation of line l:

Slope of line $k = 2$. Hence, slope of line $l = -\frac{1}{2}$.

Determine y-intercept of line l:

$$y = mx + b \rightarrow 5 = \left(-\frac{1}{2} \times -2\right) + b \rightarrow b = 4$$

Equation is $y = -\frac{x}{2} + 4$.

Determine the equation of line p: A translation of 2.5 units up of line l is

$$y = -\frac{x}{2} + (4 + 2.5) \rightarrow y = -\frac{x}{2} + 6.5$$

Determine the x-intercept of line p: Plug in $y = 0$.

$$0 = -\frac{x}{2} + 6.5 \rightarrow \frac{x}{2} = 6.5 \rightarrow x = 13$$

x-intercept $= (13, 0)$.

16. D

Determine the slope: Slope of line p is

$$-\frac{A}{B} = -\frac{2}{4} = -\frac{1}{2}$$

Since line r is perpendicular to line p, slope of line $r = 2$.

Evaluate each answer choice: Check which two points have slope $= 2$. Slope of points in answer choice D $= 2$.

17. D

Since the two lines intersect at point (2, 4), the point is on both the lines.

Determine the slope: Set up a slope equation for line m using the points (0, 0) and (2, 4).

$$\frac{4 - 0}{2 - 0} = 2$$

Since line n is perpendicular to line m, the slope of line $n = -\frac{1}{2}$.

Determine point P: Let point P be $(x, 0)$.

Set up a slope equation using the points $(x, 0)$ and $(2, 4)$ and equate to slope $= -\frac{1}{2}$.

$$\frac{4 - 0}{2 - x} = -\frac{1}{2} \rightarrow \frac{4}{2 - x} = -\frac{1}{2} \rightarrow$$

$$(4 \times 2) = -(2 - x) \rightarrow 8 = -2 + x \rightarrow x = 10$$

18. B

Determine the slope of line d:

$$-\frac{A}{B} = -\left(\frac{a}{4} \div -\frac{3}{5}\right) = -\left(\frac{a}{4} \times -\frac{5}{3}\right) = \frac{5a}{12}$$

Hence, the slope of a perpendicular line $= -\frac{12}{5a}$.

Evaluate the slope in each answer choice to match above: Only answer choice B is correct. See below.

$$-\frac{A}{B} = -\left(-\frac{3}{5a} \div -\frac{1}{4}\right) = -\left(-\frac{3}{5a} \times -\frac{4}{1}\right) = -\frac{12}{5a}$$

19. A

Determine the slope: Slope of line s is

$$-\frac{A}{B} = -\frac{4}{2} = -2$$

Since line t is perpendicular to line s, slope of line $t = \frac{1}{2}$.

Determine the y-intercept of line t: Plug the given point (4, 5) and slope $= \frac{1}{2}$ into $y = mx + b$ equation.

$$5 = \left(\frac{1}{2} \times 4\right) + b \rightarrow 5 = 2 + b \rightarrow b = 3$$

This eliminates answer choices C and D.

Determine the x-intercept of line t: Plug the slope and y-intercept into $y = mx + b$ equation and set $y = 0$.

$$0 = \frac{1}{2}x + 3 \rightarrow \frac{1}{2}x = -3 \rightarrow x = -6$$

This eliminates answer choice B.

20. B

Determine the equation of line m: y-intercept $= 10$.

$$\frac{62 - 10}{8 - 0} = \frac{52}{8} = 6.5$$

$$y = 6.5x + 10$$

Determine the equation of line n: 4.5 units down is

$$y = 6.5x + 10 - 4.5 \rightarrow y = 6.5x + 5.5$$

Standard form equation is $-6.5x + y = 5.5$.

21. D

Change from $5x$ to $5(x - 2)$ is 2 units right.

Change from -2 to 2 is 4 units up.

22. 6

Set up a slope equation and solve: Slope of line $l = 2$. Set up a slope equation using the points $(-2, -2)$ and $(2, k)$ on line l and equate to slope $= 2$.

$$\frac{k - (-2)}{2 - (-2)} = 2 \rightarrow \frac{k + 2}{4} = 2 \rightarrow k + 2 = 8 \rightarrow k = 6$$

23. 14

Set up slope equations: Set up one slope equation using the points (1, 3) and (7, 9), and another using the points $(a, 10)$ and $(4, b)$. Equate the two equations.

$$\frac{9 - 3}{7 - 1} = \frac{b - 10}{4 - a} \rightarrow \frac{6}{6} = \frac{b - 10}{4 - a} \rightarrow 1 = \frac{b - 10}{4 - a} \rightarrow$$

$$4 - a = b - 10 \rightarrow a + b = 14$$

Section 3 – Systems of Linear Equations and Inequalities

Category 9 – Systems of Linear Equations and Number of Solutions

1. <u>D</u>

$2x - 3y = -4$: $a_1 = 2$. $b_1 = -3$. $c_1 = -4$.

$-6x + 9y = 12$: $a_2 = -6$. $b_2 = 9$. $c_2 = 12$.

Evaluate the ratios:

$$\frac{a_1}{a_2} = \frac{2}{-6} = -\frac{1}{3}$$

$$\frac{b_1}{b_2} = \frac{-3}{9} = -\frac{1}{3}$$

$$\frac{c_1}{c_2} = \frac{-4}{12} = -\frac{1}{3}$$

Since all the ratios are the same, the system has infinitely many solutions.

2. <u>A</u>

The two lines in the graph intersect at one point $(-1, 2)$.

The line defined by $y = -1$ is a horizontal line that will not pass through $(-1, 2)$. Hence, the three lines don't intersect at a common point.

3. <u>B</u>

Convert the two equations to standard form:

$3x = y + 2 \rightarrow 3x - y = 2$: $a_1 = 3$. $b_1 = -1$. $c_1 = 2$.

$-y = 3x + 1 \rightarrow 3x + y = -1$: $a_2 = 3$. $b_2 = 1$. $c_2 = -1$.

Evaluate the ratios:

$$\frac{a_1}{a_2} = \frac{3}{3} = 1$$

$$\frac{b_1}{b_2} = \frac{-1}{1} = -1$$

$$\frac{c_1}{c_2} = \frac{2}{-1} = -2$$

Since the ratios of a and b are not the same, the system has one solution.

4. <u>D</u>

$\frac{1}{4}x + y = -c$: $a_1 = \frac{1}{4}$. $b_1 = 1$. $c_1 = -c$.

$x + 4y = -4c$: $a_2 = 1$. $b_2 = 4$. $c_2 = -4c$.

Evaluate the ratios:

$$\frac{a_1}{a_2} = \left(\frac{1}{4} \div 1\right) = \frac{1}{4}$$

$$\frac{b_1}{b_2} = \frac{1}{4}$$

$$\frac{c_1}{c_2} = \frac{-c}{-4c} = \frac{1}{4}$$

Since all ratios are the same, the system has infinitely many solutions.

5. <u>A</u>

Convert the top equation to standard form:

$$\frac{17}{4}x - \frac{1}{4}x + \frac{10}{3}y - \frac{1}{3}y = -\frac{1}{3} + \frac{10}{3} \rightarrow$$

$$\frac{16}{4}x + \frac{9}{3}y = \frac{9}{3} \rightarrow 4x + 3y = 3$$

$4x + 3y = 3$: $a_1 = 4$. $b_1 = 3$. $c_1 = 3$.

$\frac{1}{3}x + \frac{1}{4}y = \frac{1}{2}$: $a_2 = \frac{1}{3}$. $b_2 = \frac{1}{4}$. $c_2 = \frac{1}{2}$.

Evaluate the ratios:

$$\frac{a_1}{a_2} = \left(4 \div \frac{1}{3}\right) = 4 \times 3 = 12$$

$$\frac{b_1}{b_2} = \left(3 \div \frac{1}{4}\right) = 3 \times 4 = 12$$

$$\frac{c_1}{c_2} = \left(3 \div \frac{1}{2}\right) = 3 \times 2 = 6$$

Since ratios of a and b are the same but not the same as the ratio of c, the system has no solution.

Category 10 – Systems of Linear Equations with No Solution

1. <u>C</u>

$3x + 2y = k$: $a_1 = 3$. $b_1 = 2$. $c_1 = k$.

$9x + 6y = 12$: $a_2 = 9$. $b_2 = 6$. $c_2 = 12$.

Equate the ratios: Since the system has no solution, the value of k cannot result in the ratio of c to be same as the ratios of a and b. Equate the ratios of a and c and determine what value of k makes the ratios same.

$$\frac{a_1}{a_2} = \frac{c_1}{c_2} \rightarrow \frac{3}{9} = \frac{k}{12} \rightarrow \frac{1}{3} = \frac{k}{12} \rightarrow 1 = \frac{k}{4} \rightarrow k = 4$$

If $k = 4$, then all the ratios will be the same. For the system to have no solution $k \neq 4$.

2. <u>D</u>

Convert the two equations to the standard form:

$y = 4x + 10 \rightarrow 4x - y = -10$: $a_1 = 4$. $b_1 = -1$. $c_1 = -10$.

$-ax = -4y + 14 \rightarrow ax - 4y = -14$: $a_2 = a$. $b_2 = -4$. $c_2 = -14$.

Equate the ratios: Since the system of parallel lines has no solution, the ratios of a and b must be the same.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \rightarrow \frac{4}{a} = \frac{-1}{-4} \rightarrow a = 16$$

3. A

Evaluate each answer choice by setting up the ratios: Since the system has no solution, the ratios of a and b must be the same, but different than c.

Answer choice A is correct. See below.

$$\frac{a_1}{a_2}:\frac{b_1}{b_2}:\frac{c_1}{c_2} \rightarrow \frac{2a}{0.1a}:\frac{12b}{0.6b}:\frac{5}{0.2} \rightarrow 20:20:25$$

The ratios of a and b are same and different than c.

4. B

$0.2x + 0.3y = 0.1$: $a_1 = 0.2$. $b_1 = 0.3$. $c_1 = 0.1$.

$ax + 0.6y = 0.3$: $a_2 = a$. $b_2 = 0.6$. $c_2 = 0.3$.

Equate the ratios: Since the system has no solution, the ratios of a and b must be the same.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \rightarrow \frac{0.2}{a} = \frac{0.3}{0.6} \rightarrow 0.3a = 0.12 \rightarrow a = 0.4$$

5. D

$2x + my = n$: $a_1 = 2$. $b_1 = m$. $c_1 = n$.

$5x + 10y = 15$: $a_2 = 5$. $b_2 = 10$. $c_2 = 15$.

Equate the ratios: Since the system has no solution, the ratios of a and b must be the same but different than c.

Determine the value of m that will result in the same a and b ratios:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \rightarrow \frac{2}{5} = \frac{m}{10} \rightarrow 2 = \frac{m}{2} \rightarrow m = 4$$

This eliminates answer choices A and B.

Determine the value of n that will result in the ratios of a and b to be same as c: This cannot be the value of n. Either a or b can be equated with c.

$$\frac{a_1}{a_2} = \frac{c_1}{c_2} \rightarrow \frac{2}{5} = \frac{n}{15} \rightarrow 2 = \frac{n}{3} \rightarrow n = 6$$

Hence, for the system to have no solution $n \neq 6$.
This eliminates answer choice C.

6. D

Convert the two equations to standard form:

$$-\frac{4}{3}x + \frac{2}{3}y = \frac{3}{5} - \frac{1}{3}y \rightarrow -\frac{4}{3}x + y = \frac{3}{5}$$

$a_1 = -\frac{4}{3}$. $b_1 = 1$. $c_1 = \frac{3}{5}$.

$$ky - \frac{2}{3}x = \frac{8}{3}x + \frac{5}{3} \rightarrow -\frac{10}{3}x + ky = \frac{5}{3}$$

$a_2 = -\frac{10}{3}$. $b_2 = k$. $c_2 = \frac{5}{3}$.

Equate the ratios: Since the system has no solution, the ratios of a and b must be the same.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \rightarrow \left(-\frac{4}{3} \div -\frac{10}{3}\right) = \frac{1}{k} \rightarrow \left(-\frac{4}{3} \times -\frac{3}{10}\right) = \frac{1}{k}$$

$$\rightarrow \frac{4}{10} = \frac{1}{k} \rightarrow \frac{2}{5} = \frac{1}{k} \rightarrow \frac{5}{2} = k$$

Category 11 – Systems of Linear Equations with Infinite Solutions

1. D

$kx - 10y = c$: $a_1 = k$. $b_1 = -10$. $c_1 = c$.

$\frac{1}{2}x - \frac{1}{3}y = 2$: $a_2 = \frac{1}{2}$. $b_2 = -\frac{1}{3}$. $c_2 = 2$.

Equate the ratios:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \rightarrow \left(k \div \frac{1}{2}\right) = \left(-10 \div -\frac{1}{3}\right) = \frac{c}{2} \rightarrow$$

$$2k = 30 = \frac{c}{2}$$

Determine c: $\frac{c}{2} = 30 \rightarrow c = 60$

2. B

$mx + 6.25y = 13.75$: $a_1 = m$. $b_1 = 6.25$. $c_1 = 13.75$.

$nx + 1.25y = 2.75$: $a_2 = n$. $b_2 = 1.25$. $c_2 = 2.75$.

Equate the ratios: Setting up the ratio of c is not needed.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \rightarrow \frac{m}{n} = \frac{6.25}{1.25} = 5$$

3. B

$2ax + 5by = 50$: $a_1 = 2a$. $b_1 = 5b$. $c_1 = 50$

Evaluate each answer choice by setting up the ratios:

Answer choice A: This is incorrect. See below.

$$\frac{a_1}{a_2}:\frac{b_1}{b_2}:\frac{c_1}{c_2} \rightarrow \frac{2a}{0.2a}:\frac{5b}{0.5b}:\frac{50}{10} \rightarrow 10:10:5$$

The ratios are not the same.

Answer choice B: This is correct. See below.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \rightarrow \frac{2a}{0.5a} = \frac{5b}{1.25b} = \frac{50}{12.5} \rightarrow 4 = 4 = 4$$

No need to proceed with the remaining answer choices.

4. 6

$ax + 5y = 5$: $a_1 = a$. $b_1 = 5$. $c_1 = 5$.

$x + y = b$: $a_2 = 1$. $b_2 = 1$. $c_2 = b$.

Equate the ratios:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \rightarrow \frac{a}{1} = \frac{5}{1} = \frac{5}{b} \rightarrow a = 5 = \frac{5}{b}$$

Determine a: $a = 5$

Determine b:

$$5 = \frac{5}{b} \rightarrow 5b = 5 \rightarrow b = 1$$

Determine $a + b$: $5 + 1 = 6$

5. 1

$\sqrt{k}x + y = 2$: $a_1 = \sqrt{k}$. $b_1 = 1$. $c_1 = 2$.

$2x + cy = \sqrt{k}$: $a_2 = 2$. $b_2 = c$. $c_2 = \sqrt{k}$.

Equate the ratios:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \rightarrow \frac{\sqrt{k}}{2} = \frac{1}{c} = \frac{2}{\sqrt{k}}$$

Determine $\sqrt{k}$:

$$\frac{\sqrt{k}}{2} = \frac{2}{\sqrt{k}} \rightarrow \sqrt{k} \times \sqrt{k} = 2 \times 2 \rightarrow \sqrt{k} = 2$$

Determine c:

$$\frac{1}{c} = \frac{2}{\sqrt{k}} \rightarrow \frac{1}{c} = \frac{2}{2} \rightarrow \frac{1}{c} = 1 \rightarrow c = 1$$

6. C

$x + ky = 2$: $a_1 = 1$. $b_1 = k$. $c_1 = 2$.

$kx + ty = 2k$: $a_2 = k$. $b_2 = t$. $c_2 = 2k$.

Equate the ratios:

The values of k and t must result in the same ratios of a, b, and c. Evaluate each answer option.

Plug in $k = 1$ and $t = 1$: The ratios are

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \rightarrow \frac{1}{1} = \frac{1}{1} = \frac{2}{2 \times 1} \rightarrow 1 = 1 = 1$$

This is true for infinitely many solutions.

Plug in $k = 2$ and $t = 4$: The ratios are

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \rightarrow \frac{1}{2} = \frac{2}{4} = \frac{2}{2 \times 2} \rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$$

This is true for infinitely many solutions.

Plug in $k = 3$ and $t = 6$: The ratios are

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \rightarrow \frac{1}{3} = \frac{3}{6} = \frac{2}{2 \times 3} \rightarrow \frac{1}{3} = \frac{1}{2} = \frac{1}{3}$$

This is not true for infinitely many solutions since all ratios are not the same.

Hence, the correct answer choice is C.

Category 12 – Systems of Linear Equations with One Solution

1. C

Determine the approach: Since it is given that $x = 4y$, the easiest approach is to substitute $4y$ for x in the equation $-5x + 2y = -54$ and solve for y.

Solve y:

$$-5(4y) + 2y = -54 \rightarrow -20y + 2y = -54 \rightarrow$$
$$-18y = -54 \rightarrow y = 3$$

2. C

Read the intersection point on the graph: The coordinates of intersection are $(1, 1)$.

3. D

Determine the approach: Adding the two equations will give the value of $8x - 2y$. Multiplying this by 10 will give the value of $80x - 20y$.

Solve for $80x - 20y$:

$$4x + y = 7$$
$$4x - 3y = 12$$
$$\overline{8x - 2y = 19}$$
$$(8x - 2y = 19) \times 10 \rightarrow 80x - 20y = 190$$

4. C

Determine the approach: If the equation $2x - 3y = 4$ is multiplied by 3 and equation $3x - 2y = 6$ is multiplied by 2, then both the equations will have $6x$ that can be canceled.

Equation 1: $(2x - 3y = 4) \times 3 \rightarrow 6x - 9y = 12$

Equation 2: $(3x - 2y = 6) \times 2 \rightarrow 6x - 4y = 12$

Determine y: Subtract equation 2 from equation 1.

$$\begin{array}{l} 6x - 9y = 12 \\ -(6x - 4y = 12) \end{array} \longrightarrow \begin{array}{r} \cancel{6x} - 9y = 12 \\ -\cancel{6x} + 4y = -12 \\ \hline -5y = 0 \rightarrow y = 0 \end{array}$$

Determine x: Substitute $y = 0$ in either equation.

$$2x - 3y = 4 \rightarrow 2x - 0 = 4 \rightarrow x = 2$$

5. A

Convert the two equations to standard form:

$$\frac{1}{4}x - by = 7 - \frac{15}{4}x \rightarrow \frac{1}{4}x + \frac{15}{4}x - by = 7 \rightarrow$$
$$4x - by = 7$$
$$-x + 6y = 2 + 7x \rightarrow -8x + 6y = 2$$

$4x - by = 7$: $a_1 = 4$. $b_1 = -b$. $c_1 = 7$.

$-8x + 6y = 2$: $a_2 = -8$. $b_2 = 6$. $c_2 = 2$.

Equate the ratios: Since the system has one solution, the ratios of a and b cannot be same. Equate the ratios of a and b and determine what value of b makes the ratios the same. This cannot be the value of b.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \rightarrow \frac{4}{-8} = \frac{-b}{6} \rightarrow -\frac{1}{2} = -\frac{b}{6} \rightarrow b = 3$$

Hence, for the system to have one solution $b \neq 3$.

6. 4

Determine the approach: Since the point $(3, 5)$ is the solution to the system, it is the intersection point and lies on the lines graphed by both the equations. Plug these values of x and y in both the equations and set up a system to solve for k.

Plug $x = 3$ and $y = 5$ in both the equations:

$$-3j(3) + 2.5k(5) = 23 \rightarrow -9j + 12.5k = 23$$
$$1.5j(3) + 7.5k(5) = 163.5 \rightarrow 4.5j + 37.5k = 163.5$$

Solve for k: If the equation $4.5j + 37.5k = 163.5$ is multiplied by 2, then both the equations will have $9j$ that can be canceled.

$$\begin{array}{r} -9j + 12.5k = 23 \\ 2(4.5j + 37.5k = 163.5) \end{array} \longrightarrow \begin{array}{r} -\cancel{9j} + 12.5k = 23 \\ \cancel{9j} + 75k = 327 \\ \hline 87.5k = 350 \\ k = 4 \end{array}$$

Category 13 – Systems of Linear Inequalities

1. C

Plug points from the answer choices: Start with choice B.

Answer choice B: Plug the point $(1, -2)$ into one of the inequalities.

Check $y > -2x + 4$:

$$-2 > (-2 \times 1) + 4 \rightarrow -2 > -2 + 4 \rightarrow -2 > 2$$

This evaluation is false. This eliminates answer choice B.

Answer choice C: Plug in the point (1, 3).

Check $y > -2x + 4$:

$$3 > (-2 \times 1) + 4 \rightarrow 3 > -2 + 4 \rightarrow 3 > 2$$

This evaluation is true.

Check $y < 3x + 1$:

$$3 < (3 \times 1) + 1 \rightarrow 3 < 4$$

This evaluation is true. Answer choice C is correct.

2. B

Graph the inequalities based on the slope, y-intercept, and inequality symbol.

The overlapping region is in quadrants II and III.

3. D

Evaluate the two inequalities:

$2x > -17$ can be rewritten to have the same expression as in $y > 2x + 29$ by adding 29 to both sides. See below.

$$2x + 29 > -17 + 29 \rightarrow 2x + 29 > 12$$

Hence, the two inequalities in the system are $y > 2x + 29$ and $2x + 29 > 12$. Using the transitive property of inequalities, they can be written as

$$y > 12$$

4. D

Rearrange inequalities.

$$-x + y \geq -a \rightarrow y \geq x - a$$

$$-2x - y \geq -b \rightarrow -y \geq 2x - b \rightarrow y \leq -2x + b$$

Compare the slope and inequality symbol (y-intercepts are same in all the graphs):

$y \geq x - a$:

The slope is positive. Since the inequality has ≥ symbol, the solution set will be on and above the line. This eliminates answer choices B and C that have a line with a positive slope but solution below the line.

$y \leq -2x + b$:

The slope is negative. Since the inequality has ≤ symbol, the solution set will be below the line. This eliminates answer choice A that has a line with a negative slope but solution above the line.

Category 14 – Equivalent and Nonequivalent Linear Expressions

1. B

The left-side expression does not have the variable x. Hence, the equation has one solution.

2. 10

Equate the constants: The expressions on both sides of the equation must be equal.

$$20 = 2b \rightarrow b = 10$$

3. A

Equate coefficients and constants: The expressions on both sides of the equation must be equal.

Since there is no x variable in the right-side expression, $b = 0$.

4. 4

Equate coefficients and constants: The expressions on both sides of the equation must be equal.

Simplify the left-side expression and add the constants.

$$5x + 5 + 3a + 1 = 5x + 18 \rightarrow$$

$$5x + 3a + 6 = 5x + 18$$

The coefficients on both sides are equal. Equate the constants and determine the value a that makes constants on both sides of the equation equal.

$$3a + 6 = 18 \rightarrow 3a = 12 \rightarrow a = 4$$

5. 4

Equate coefficients: The coefficient of x in the two expressions must be equal.

Factor x in the right-side expression.

$$1 = -14ax + 56x \rightarrow 1 = (-14a + 56)x$$

Equate the coefficients and determine the value of a that makes coefficients on both sides of the equation equal.

The coefficient in the left-side expression equates to 0. Hence,

$$-14a + 56 = 0 \rightarrow 14a = 56 \rightarrow a = \frac{56}{14} = 4$$

6. B

Equate coefficients: The coefficient of x in the two expressions must be equal.

$$0.2kx = \frac{x+9}{5} \ \rightarrow \ 0.2kx = \frac{x}{5} + \frac{9}{5}$$

Equate the coefficients and determine the value k that makes coefficients on both sides of the equation equal.

$$0.2k = \frac{1}{5} \ \rightarrow \ k = \frac{1}{5 \times 0.2} = \frac{1}{1} = 1$$

7. 7.35

Equate coefficients: The coefficient of x in the two expressions must be equal. (Since c is less than 0, $-\frac{1}{9}c$ will be positive for any value of c, hence, cannot be same as $-\frac{48}{15}$).

Simplify the left-side expression.

$$\frac{1}{9}(4kx) - \frac{1}{9}c = \frac{49}{15}x - \frac{48}{15}$$

Equate the coefficients and determine the value of k that makes coefficients on both sides of the equation equal.

$$\frac{4k}{9} = \frac{49}{15} \ \rightarrow \ 4k \times 15 = 49 \times 9 \ \rightarrow \ 60k = 441 \ \rightarrow$$
$$k = 7.35$$

8. D

Evaluate coefficients and constants:

$$-18(12x - 5) = 12(7.5 - 18x) \ \rightarrow$$
$$-216x + 90 = 90 - 216x \ \rightarrow$$
$$-216x + 90 = -216x + 90$$

The coefficients and constants in the two expressions are the same. Hence, the equation has infinitely many solutions.

Section 3 – Drill

1. D

$3x - 2y = 9 : a_1 = 3.\ b_1 = -2.\ c_1 = 9.$

$9x - 6y = 27 : a_2 = 9.\ b_2 = -6.\ c_2 = 27.$

Evaluate the ratios:

$$\frac{a_1}{a_2} : \frac{b_1}{b_2} : \frac{c_1}{c_2} := \frac{3}{9} : \frac{-2}{-6} : \frac{9}{27} = \frac{1}{3} : \frac{1}{3} : \frac{1}{3}$$

Since all the ratios are the same, the system has infinitely many solutions.

2. 4

Equate coefficients and constants: The expressions on both sides of the equation must be equal.

Equate the constants:

$$3k = 12 \ \rightarrow \ k = 4$$

3. B

$6x + 36y = 54 : a_1 = 6.\ b_1 = 36.\ c_1 = 54.$

$x + 6y = c : a_2 = 1.\ b_2 = 6.\ c_2 = c.$

Equate the ratios and determine c: Since the system has infinitely many solutions, all the ratios are the same. Equate the ratio of either a or b with the ratio of c.

$$\frac{a_1}{a_2} = \frac{c_1}{c_2} \ \rightarrow \ \frac{6}{1} = \frac{54}{c} \ \rightarrow \ 6c = 54 \ \rightarrow \ c = 9$$

4. A

Convert the top equation to standard form:

$$\frac{b}{5}x = 4 + 2y \ \rightarrow \ \frac{b}{5}x - 2y = 4$$

$\frac{b}{5}x - 2y = 4 : a_1 = \frac{b}{5}.\ b_1 = -2.\ c_1 = 4.$

$3x - 12y = 7 : a_2 = 3.\ b_2 = -12.\ c_2 = 7.$

Equate the ratios: Since the system has no solution, the ratios of a and b must be the same.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \ \rightarrow \ \left(\frac{b}{5} \div 3\right) = \frac{-2}{-12} \ \rightarrow \ \frac{b}{15} = \frac{1}{6} \ \rightarrow$$
$$6b = 15 \ \rightarrow \ b = \frac{15}{6} = \frac{5}{2} = 2.5$$

5. 20/8, 10/4, 5/2, or 2.5

Decide the approach: Adding the two equations will give the value of $8x - 8y$. Dividing this by 8 will give the value of $x - y$.

Solve for $x - y$: Add both equations

$$2x - 5y = 11$$
$$6x - 3y = 9$$
$$\overline{8x - 8y = 20}$$

$$\frac{8x - 8y}{8} = \frac{20}{8} \ \rightarrow \ x - y = \frac{20}{8} = 2.5$$

6. <u>3</u>

$(a+4)x+4y=4$: $a_1=a+4$. $b_1=4$. $c_1=4$.

$3x+by=3$: $a_2=3$. $b_2=b$. $c_2=3$.

Equate the ratios:

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \rightarrow \frac{a+4}{3}=\frac{4}{b}=\frac{4}{3}$$

Determine a:

$$\frac{a+4}{3}=\frac{4}{3} \rightarrow a+4=4 \rightarrow a=0$$

Determine b:

$$\frac{4}{3}=\frac{4}{b} \rightarrow b=3$$

Determine $a+b$: $0+3=3$

7. <u>A</u>

Equate coefficients and constants: The expressions on both sides of the equation must be equal.

Since there is no x variable in the right-side expression, the coefficient of x in the left-side expression must be 0.

8. <u>C</u>

Compare the slope and inequality symbol: Both the inequalities have y-intercept= 1.

Both inequalities have solution set above the line. This eliminates answer choices A and B.

Note that in the remaining answer choices C and D, one of the inequalities has a negative y, and answer choice C has one inequality with $2y$.

Rearrange these inequalities before comparison.

Answer choice C:

$$2y \geq 3x+2 \rightarrow y \geq 1.5x+1$$

$$-y \leq 2x-1 \rightarrow y \geq -2x+1$$

Answer choice D: $-y \leq 2x+1 \rightarrow y \geq -2x-1$

y-intercept $=-1$ eliminates answer choice D.

9. <u>C</u>

$x-y=n$: $a_1=1$. $b_1=-1$. $c_1=n$.

$-mx+2y=12$: $a_2=-m$. $b_2=2$. $c_2=12$.

Equate the ratios: For the system to have no solution, the values of m and n must result in the same ratios of a and b but not c. Evaluate each answer option.

<u>Plug in $m=-2$ and $n=2$</u>: The ratios are

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2} \rightarrow \frac{1}{-(-2)}=\frac{-1}{2}=\frac{2}{12} \rightarrow \frac{1}{2}=-\frac{1}{2}=\frac{1}{6}$$

The ratios of a and b are not same. This is false.

<u>Plug in $m=2$ and $n=-6$</u>: The ratios are

$$\frac{1}{-2}=\frac{-1}{2}=\frac{-6}{12} \rightarrow -\frac{1}{2}=-\frac{1}{2}=-\frac{1}{2}$$

All ratios are the same. This is false.

<u>Plug in $m=2$ and $n=6$</u>: The ratios are

$$\frac{1}{-2}=\frac{-1}{2}=\frac{6}{12} \rightarrow -\frac{1}{2}=-\frac{1}{2}=\frac{1}{2}$$

The ratios of a and b are the same but not c. This is true.

10. <u>B</u>

Read the intersection point: (2, 2).

The system will have zero solution if the line of the equation $2x-y=2$ does not intersect at (2, 2) and one solution if it does.

Plug (2, 2) into the equation $2x-y=2$ to determine if the point is on the line:

$2x-y=2 \rightarrow (2\times 2)-(1\times 2)=2 \rightarrow 2=2$. This is true. Hence, (2, 2) is on the line.

All lines intersect at (2, 2). The system has one solution.

11. <u>A</u>

Plug $x=-10$ into each inequality and evaluate the possible values of y.

$y>\frac{x}{4}-2$: Plug $x=-10$.

$y>\frac{-10}{4}-2 \rightarrow y>-2.5-2 \rightarrow y>-4.5$

$3x+15y<18$: Plug $x=-10$.

$$(3\times -10)+15y<18 \rightarrow -30+15y<18 \rightarrow$$

$$15y<18+30 \rightarrow 15y<48 \rightarrow y<3.2$$

Hence, $-4.5<y<3.2$.

Evaluate each answer choice for this condition. Only answer choice A does not meet it.

12. <u>10</u>

Equate coefficients: The coefficients of x on both sides of the equation must be equal for the equation to have no solution.

Simplify the right-side expression.

$$11x+50=4x+kx-3x \rightarrow 11x+50=x+kx$$

$$11x+50=(1+k)x$$

Equate the coefficients and determine the value of k that makes coefficients on both sides of the equation equal.

$$11=1+k \rightarrow k=11-1=10$$

13. <u>C</u>

Convert the two equations to standard form:

$$5x+3y=a+3x \rightarrow 2x+3y=a$$

$$-\frac{4}{3}x-\frac{a}{6}=-\frac{1}{2}y-\frac{5}{3}x \rightarrow \frac{1}{3}x+\frac{1}{2}y=\frac{a}{6}$$

$2x+3y=a$: $a_1=2$. $b_1=3$. $c_1=a$.

$\frac{1}{3}x+\frac{1}{2}y=\frac{a}{6}$: $a_2=\frac{1}{3}$. $b_2=\frac{1}{2}$. $c_2=\frac{a}{6}$.

Evaluate the ratios:

$$\frac{a_1}{a_2}=\left(2\div\frac{1}{3}\right)=2\times 3=6$$

$$\frac{b_1}{b_2}=\left(3\div\frac{1}{2}\right)=3\times 2=6$$

$$\frac{c_1}{c_2}=\left(a\div\frac{a}{6}\right)=a\times\frac{6}{a}=6$$

Since all the ratios are the same, the system is for the same line.

14. <u>112</u>

Decide the approach: Adding the two equations will give the value of $6(x+5)$. Multiplying this by 4, will give the value of $24(x+5)$.

Solve for $24(x+5)$: Add the two equations.

$$3(x+5) + \cancel{5(y-2)} = 455$$
$$3(x+5) - \cancel{5(y-2)} = -427$$
$$6(x+5) \qquad = 28$$
$$(6(x+5)) = 28) \times 4 \ \rightarrow \ 24\,(x+5) = 112$$

15. <u>B</u>

Divide both sides of the inequality $-5y > -25x + 2.5$ by -5 and flip the inequality symbol.

$$\frac{-5y}{-5} < \frac{-25}{-5}x + \frac{2.5}{-5} \ \rightarrow \ y < 5x - 0.5$$

Evaluate the two inequalities:

$5x < 3$ can be rewritten to have the same expression as in $y < 5x - 0.5$ by subtracting -0.5 from both sides. See below.

$$5x - 0.5 < 3 - 0.5 \ \rightarrow \ 5x - 0.5 < 2.5$$

Hence, the two inequalities in the system are $y < 5x - 0.5$ and $5x - 0.5 < 2.5$. Using the transitive property of inequalities, they can be written as

$$y < 2.5$$

16. <u>B</u>

$3x - 4y = 5$: $a_1 = 3$. $b_1 = -4$. $c_1 = 5$

Evaluate each answer choice: Set up the ratio using the given equation and the equation from each answer choice. The equation of the correct answer choice will result in the same ratio of a and b that is different from that of c.

Answer choice A: This is incorrect. See below.

$$\frac{a_1}{a_2}:\frac{b_1}{b_2}:\frac{c_1}{c_2} \ \rightarrow \ \frac{3}{\left(-\frac{1}{4}\right)}:\frac{-4}{\left(-\frac{1}{3}\right)}:\frac{5}{6} \ \rightarrow \ -12: 12: \frac{5}{6}$$

The ratios of a and b are not the same.

Answer choice B: This is correct. See below.

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \ \rightarrow \ \frac{3}{\left(-\frac{1}{2}\right)}:\frac{-4}{\left(\frac{2}{3}\right)}:\frac{5}{6} \ \rightarrow \ -6: -6: \frac{5}{6}$$

The ratios of a and b are the same and different than c. No need to proceed with the remaining answer choices.

17. <u>C</u>

Decide the approach: Rearrange the equation $5y = -x + 3$ so the variables x and y can be aligned in the two equations.

$$5y = -x + 3 \ \rightarrow \ x + 5y = 3$$

If the above equation is multiplied by 2, then both the equations will have $2x$ that can be canceled.

Equation 1: $(x + 5y = 3) \times 2 \ \rightarrow \ 2x + 10y = 6$

Equation 2: $2x + 3y = -8$

Determine y: Subtract equation 2 from equation 1.

$$\begin{array}{l} 2x + 10y = 6 \\ -(2x + 3y = -8) \end{array} \longrightarrow \begin{array}{l} \cancel{2x} + 10y = 6 \\ -\cancel{2x} - 3y = 8 \end{array}$$
$$7y = 14 \ \rightarrow \ y = 2$$

Determine x: Substitute $y = 2$ in any of the equations.

$$x + (5 \times 2) = 3 \ \rightarrow \ x + 10 = 3 \ \rightarrow \ x = -7$$

18. <u>C</u>

Evaluate the slope of the two equations in each answer choice as $-\frac{A}{B}$:

The slope of one equation should be the negative reciprocal of the other. Applying the tip mentioned in Category 3, answer choices A and D can be eliminated.

In answer choice A, both the equations have the subtraction operator for A and B. Hence, the slope of both the equations is negative.

In answer choice D, both the equations have the addition operator for A and B. Hence, the slope of both the equations is negative.

Evaluate the remaining answer choices B and C.

Answer choice B:

Top equation:

$$-\frac{A}{B} = -\left(-\frac{9}{7a} \div \frac{3}{14b}\right) = -\left(-\frac{9}{7a} \times \frac{14b}{3}\right) = \frac{6b}{a}$$

Bottom equation:

$$-\frac{A}{B} = -\left(\frac{2a}{3} \div b\right) = -\left(\frac{2a}{3} \times \frac{1}{b}\right) = -\frac{2a}{3b}$$

This eliminates answer choice B.

Hence, answer choice C must be correct. See below. The slopes are negative reciprocals.

Top equation:

$$-\frac{A}{B} = -\left(\frac{9}{7a} \div -\frac{3}{14b}\right) = -\left(\frac{9}{7a} \times -\frac{14b}{3}\right) = \frac{6b}{a}$$

Bottom equation:

$$-\frac{A}{B} = -\left(\frac{2a}{3} \div 4b\right) = -\left(\frac{2a}{3} \times \frac{1}{4b}\right) = -\frac{a}{6b}$$

Section 4 – Word Problems on Linear Equations and Inequalities

Category 15 – Word Problems on Linear Equations with One Variable

1. A

Determine the constant: Entry fee = \$10. This eliminates answer choices C and D.

Determine the unknown number: h hours at \$5 per hour $= 5h$. This eliminates answer choice B.

2. B

Determine the constant: One-time fee = \$25. This eliminates answer choices A and C.

Determine the unknown number: t hours at \$20 per hour $= 20t$. This eliminates answer choice D.

3. D

Determine the constant: Initial balance $= d$. This eliminates answer choices A and C.

Determine the unknown number: \$100 per month in m months $= 100m$. This eliminates answer choice B.

4. D

Determine the constant: One-time rental fee = \$75.

Determine the unknown number: 4 days at \$55 per day $= 4 \times 55 = \$220$.

Solve:

$$\text{total cost} = 75 + 220 = \$295$$

5. D

Determine the constant: Entrance fee = \$8. This eliminates answer choices A and B.

Determine the unknown number: r rides at \$1.50 per ride $= 1.50r$. This eliminates answer choice C.

6. C

Determine the constant: Fixed service fee $= s$.

Determine the unknown number: 3 hours at p dollars per hour $= 3p$.

Solve:

$$s + 3p = \$360 \;\rightarrow\; s = 360 - 3p$$

7. D

Initial number of research papers $= 30$.

Determine the unknown number:

Unknown number = number of research papers × 40.

The number of minutes required to complete the remaining $30 - r$ research papers is

$$(30 - r) \times 40 \text{ minutes}$$

Convert to hours.

$$\frac{(30 - r) \times 40}{60} \;\rightarrow\; \frac{2}{3}(30 - r)$$

8. B

Initial number of tickets $= x$.

Determine the unknown number:

The number of tickets remaining after the first day is

$$x - 400$$

Solve: Number of remaining tickets T to be sold each day for the next 4 days is

$$\frac{x - 400}{4} \;\rightarrow\; \frac{x}{4} - \frac{400}{4} \;\rightarrow\; \frac{x}{4} - 100$$

9. B

Determine the constant: Initial saved amount $= d$ dollars.

Determine the unknown number: \$150 per month in m months $= 150m$.

Solve: Total needed = \$1,500

$$\text{total} = d + 150m \;\rightarrow\; \$1{,}500 = d + 150m$$

Rearrange the equation to express in terms of m.

$$150m = 1{,}500 - d \;\rightarrow\; m = \frac{1{,}500 - d}{150} \;\rightarrow$$

$$m = \frac{1{,}500}{150} - \frac{d}{150} = 10 - \frac{d}{150}$$

10. A

Plug $h = 20$ into the given equation.

$$g = 0.003h + 0.2 \;\rightarrow\; g = (0.003 \times 20) + 0.2 \;\rightarrow$$

$$g = 0.06 + 0.2 \;\rightarrow\; g = 0.26$$

11. C

Initial number of notebooks $= 255$.

Determine the unknown number: Let number of weeks $= x$. Number of notebooks in x weeks $= 15x$.

75 notebooks should remain in x weeks. Hence,

$$255 - 15x = 75 \;\rightarrow\; 15x = 180 \;\rightarrow\; x = \frac{180}{15} = 12$$

12. D

Determine the unknown number: Total trips $= 14$.

Number of trips from Station A to Station C $= x$. Hence, number of trips from Station A to Station B $= 14 - x$.

Solve:

Total cost from Station A to Station C $= 4.5x$.

Total cost from Station A to Station B $= 3.25(14 - x) = 45.5 - 3.25x$.

$$\text{total cost} = 4.5x + 45.5 - 3.25x = 45.5 + 1.25x$$

Category 16 – Word Problems on Linear Equations with Two Variables

1. A

Determine the unknown number 1 (morning shift): m hours at \$15 per hour $= 15m$. This eliminates answer choices B and D.

Determine the unknown number 2 (night shift): n hours at \$25 per hour $= 25n$. This eliminates answer choice C.

2. C

Determine the unknown number 1 (pounds of paper): m.

Determine the unknown number 2 (pounds of cans): $n = 582$.

Solve for m: Substitute $n = 582$ in the given equation.

$$m + 582 = 1{,}277 \rightarrow m = 1{,}277 - 582 = 695$$

3. B

Determine the unknown number 1 (snow cones): 4 snow cones for c dollars each $= 4c$.

Determine the unknown number 2 (frozen yogurt): 7 frozen yogurts for $c + 1$ dollars each $= 7(c + 1)$.

Solve: Total cost = 4 snow cones + 7 yogurts.

$$40 = 4c + 7(c + 1) \rightarrow 40 = 4c + 7c + 7 \rightarrow$$
$$11c = 33 \rightarrow c = 3$$

4. D

Determine the unknown number 1 (working at restaurant): x hours at \$20 per hour $= 20x$. This eliminates answer choices A and B.

Determine the unknown number 2 (working at hotel): 5 hours at \$35 per hour $= 35 \times 5 = \$175$. This eliminates answer choice C.

5. 4

Determine the unknown number 1 (Crane A): 100 boxes lifted per hour for 2 hours $= 100 \times 2 = 200$.

Determine the unknown number 2 (Crane B): Let the number of hours $= x$. Hence, 75 boxes lifted per hour for x hours $= 75x$.

Solve: Total boxes lifted = Crane A + Crane B.

$$500 = 200 + 75x \rightarrow 300 = 75x \rightarrow x = 4$$

6. 3

Determine the unknown number 1 (painter): 4 hours at \$80 per hour $= 4 \times 80 = \$320$.

Determine the unknown number 2 (landscaper): a hours at \$50 per hour $= 50a$.

Solve: Total earnings = painter + landscaper.

$$470 = 320 + 50a \rightarrow 50a = 150 \rightarrow a = 3$$

Category 17 – Word Problems on Interpretation of Linear Equations

1. D

Determine the components of the equation:

2 = Average number of research papers Anika reviews each day of the month.

58 = Initial number of research papers Anika receives at the beginning of each month.

R = Ending number of research papers at the end of each day of the month.

2. A

Determine the components of the equation:

0.0054 = Average increase in the growth of the plant, in inches, in response to each hour of sunlight.

15 = Initial size, in inches, of the plant before exposure to sunlight.

3. B

Determine the components of the equation:

72 = Total acres of land.

x = average number of homes to be built on 1 acre of land.

y = average number of homes to be built on 1.5 acres of land.

4. C

Determine the components of the equation:

a = Total feet of rope bought.

b = Number of jump ropes made.

$9b$ = Total feet of rope used to make b jump ropes.

8 = Remaining feet of rope after making b jump ropes.

5. D

Determine the components of the equation:

a = Number of 5-pound boxes.

b = Number of 12-pound boxes.

$a + b$ = Total number of 5-pound and 12-pound boxes.

Category 18 – Word Problems on Linear System of Equations

1. B

Identify the two variables:

Number of spiral notebooks $= n$.

Number of pocket folders $= p$.

Determine the system of equations:

Equation 1: Total of notebooks and pocket folders $= 78$.

$$n + p = 78$$

This eliminates answer choices C and D.

Equation 2: Price of each $n = \$3$, and price of each $p = \$0.50$. Total cost $= \$164$.

$$3n + 0.5p = 164$$

This eliminates answer choice A.

2. D

Identify the two variables:

Number of shirts $= s$.

Number of hats $= h$.

Determine the system of equations:

Equation 1: Total of shirts and hats $= 10$.

$$s + h = 10$$

This eliminates answer choices A and B.

Equation 2: Price of each $s = \$20$. Price of each $h = \frac{1}{2}s = \frac{20}{2} = \$10$. Total cost $= \$140$.

$$20s + 10h = 140$$

This eliminates answer choice C.

3. 30

Identify the two variables:

Variables a and b are two different numbers.

Determine the system of equations:

Equation 1:

$$a + b = 110$$

Equation 2:

$$3a + b = 170$$

Solve the system for a:

$$\begin{array}{l} 3a + b = 170 \\ -(a + b = 110) \end{array} \longrightarrow \begin{array}{l} 3a + \cancel{b} = 170 \\ -a - \cancel{b} = -110 \\ \hline 2a \quad\quad = 60 \rightarrow a = 30 \end{array}$$

4. C

Identify the two variables:

Let number of fish tacos $= x$.

Let number of chicken tacos $= y$.

Determine the system of equations:

Equation 1: Total number of tacos $= 14$.

$$x + y = 14$$

Equation 2: Price of each $x = \$6$, and price of each $y = \$4$. Total cost of tacos $= \$72$.

$$6x + 4y = 72$$

Solve the system for x (fish tacos):

$$\begin{array}{l} 6x + 4y = 72 \\ -4(x + y = 14) \end{array} \longrightarrow \begin{array}{l} 6x + \cancel{4y} = 72 \\ -4x - \cancel{4y} = -56 \\ \hline 2x \quad\quad = 16 \rightarrow x = 8 \end{array}$$

5. 15

Identify the two variables:

Let Shannon's present age $= x$.

Hence, Shannon's age 3 years ago $= x - 3$.

Let Tina's present age $= y$.

Hence, Tina's age 3 years ago $= y - 3$.

Determine the system of equations:

Equation 1: Total of present ages $= 42$.

$$x + y = 42$$

Equation 2: Three years ago Tina's age $(y - 3)$ was twice of Shannon's age $(x - 3)$.

$$(y - 3) = 2(x - 3) \rightarrow$$

$$y - 3 = 2x - 6 \rightarrow 2x - y = 3$$

Solve the system for x (Shannon's present age):

$$\begin{array}{l} x + \cancel{y} = 42 \\ 2x - \cancel{y} = 3 \\ \hline 3x \quad = 45 \rightarrow x = 15 \end{array}$$

6. D

Identify the two variables:

Number of hours through backroads $= b$.

Number of hours on highway $= h$.

Determine the system of equations:

Equation 1: Equation $b + h = 5$ is given in the question.

Equation 2: Total miles $= 205$.

Since the speed on backroads is 25 miles in an hour, miles in b hours $= 25b$.

Since the speed on highway is 65 miles in an hour, miles in h hours $= 65h$.

$$25b + 65h = 205$$

This eliminates answer choices A, B, and C.

7. <u>24</u>

Identify the two variables:

Let number of dimes $= x$.

Let number of quarters $= y$.

Determine the system of equations:

Equation 1: Total number of coins $= 40$.

$$x + y = 40$$

Equation 2: Each dime = \$0.10. Each quarter = \$0.25. Total value = \$7.60.

$$0.10x + 0.25y = 7.60 \rightarrow 10x + 25y = 760$$

Solve the system for y (quarters):

$$\begin{array}{lcr} 10x + 25y = 760 & & \cancel{10x} + 25y = 760 \\ -10(x + y = 40) & \longrightarrow & -\cancel{10x} - 10y = -400 \\ \hline & & 15y = 360 \\ & & y = 24 \end{array}$$

8. <u>10</u>

Identify the two variables:

Number of muffin trays $= a$.

Number of cupcake trays $= b$.

Determine the system of equations:

Equation 1: Total number of trays $= 18$.

$$a + b = 18$$

Equation 2: Price of each $a = \$10$, and price of each $b = \$6$. Total cost of all trays = \$148.

$$10a + 6b = 148$$

Solve the system for a (tray of muffins):

$$\begin{array}{lcr} 10a + 6b = 148 & & 10a + \cancel{6b} = 148 \\ -6(a + b = 18) & \longrightarrow & -6a - \cancel{6b} = -108 \\ \hline & & 4a \quad = 40 \rightarrow a = 10 \end{array}$$

9. <u>A</u>

Identify the two variables:

Let number of rooms with 2 beds $= x$.

Let number of rooms with 3 beds $= y$.

Determine the system of equations:

Equation 1: Total number of rooms $= 55$.

$$x + y = 55$$

Equation 2: Each room with 2 beds (x) will have 2 students, and each room with 3 beds (y) will have 3 students. Total number of students $= 125$.

$$2x + 3y = 125$$

Solve the system for y (rooms with 3 beds):

$$\begin{array}{lcr} 2x + 3y = 125 & & \cancel{2x} + 3y = 125 \\ -2(x + y = 55) & \longrightarrow & -\cancel{2x} - 2y = -110 \\ \hline & & y = 15 \end{array}$$

10. <u>B</u>

Identify the two variables:

Let cost of each box of crayons $= x$.

Let cost of each box of pencils $= y$.

Determine the system of equations:

Equation 1: Cost of 6 boxes of crayons and 5 boxes of pencils = \$25.50.

$$6x + 5y = 25.50$$

Equation 2: Cost of $6 - 2 = 4$ boxes of crayons and $5 + 3 = 8$ boxes of pencils = \$24.

$$4x + 8y = 24$$

Solve the system for y (pencils):

$$\begin{array}{lcr} 6(4x + 8y = 24) & & \cancel{24x} + 48y = 144 \\ -4(6x + 5y = 25.50) & \longrightarrow & -\cancel{24x} - 20y = -102 \\ \hline & & 28y = 42 \rightarrow y = 1.5 \end{array}$$

11. <u>250</u>

Identify the two variables:

Let number of tickets sold to ages 12 and below $= c$.

Let number of tickets sold to ages 13 and above $= a$.

Determine the system of equations:

Equation 1: Total tickets sold $= 400$.

$$a + c = 400$$

Equation 2: Price of each $a = \$14$, and price of each $c = \$10$. Total tickets sold for = \$4,600.

$$14a + 10c = 4{,}600$$

Solve the system for c (sold to ages 12 and below):

$$\begin{array}{lcr} 14(a + c = 400) & & \cancel{14a} + 14c = 5{,}600 \\ -(14a + 10c = 4{,}600) & \longrightarrow & -\cancel{14a} - 10c = -4{,}600 \\ \hline & & 4c = 1{,}000 \\ & & c = 250 \end{array}$$

12. <u>D</u>

Identify the two variables:

Let the number of 20-ounce bottles $= a$.

Let the number of 50-ounce bottles $= b$.

Determine the system of equations:

Equation 1: Total number of bottles $= 90$.

$$a + b = 90$$

Equation 2: Price of each $a = \$20$ and price of each $b = \$42$. Use the process of elimination. First determine the lowest value of b using the lower total = \$2,900.

$$20a + 42b = 2{,}900$$

Solve the system for b:

$$\begin{array}{lcr} 20a + 42b = 2{,}900 & & \cancel{20a} + 42b = 2{,}900 \\ -20(a + b = 90) & \longrightarrow & -\cancel{20a} - 20b = -1{,}800 \\ \hline & & 22b = 1{,}100 \\ & & b = 50 \end{array}$$

Hence, the value of b is 50 or higher. 50 is not an answer choice. Only answer choice D is higher than 50.

Category 19 – Word Problems on Linear Inequalities

1. <u>B</u>

Determine the variables:

Number of hot dogs $= d$.

Determine the conditional relationship:

Cost of daily stand = $68.

Cost of making d hot dogs $= 1.5d$.

Earnings from d hot dogs $= 4d$.

Daily cost of stand + daily cost of making d hot dogs must be less than or equal to daily earnings from selling d hot dogs.

$$68 + 1.5d \leq 4d \;\rightarrow\; 68 \leq 4d - 1.5d \;\rightarrow\; 68 \leq 2.5d$$

2. <u>A</u>

Determine the variables:

Number of cards given to brother $= a$.

Number of cards given to sister $= b$.

Determine the conditional relationship:

Total cards given $= a + b$.

Total cards left $= 15 - (a + b)$.

Total cards left is at least 5.

$$15 - (a + b) \geq 5 \;\rightarrow\; 15 - a - b \geq 5$$

3. <u>C</u>

Determine the variables:

Number of soft pretzels $= p$.

Number of water bottles $= b$.

Determine the conditional relationship:

Cost of p soft pretzels $= 3p$.

Cost of b water bottles $= 2b$.

Cost of p soft pretzels + cost of b water bottles must be less than or equal to 200.

$$3p + 2b \leq 200 \;\rightarrow\; 200 \geq 3p + 2b$$

4. <u>D</u>

Determine the variable:

Amount in bank account, in dollars $= s$.

Determine the conditional relationship: Monthly deposits must be greater than or equal to $100 but less than or equal to $150.

$$100 \leq \text{monthly deposit} \leq 150.$$

In 12 monthly deposits, all possible amounts are

$$(100 \times 12) \leq \text{deposits in 12 months} \leq (150 \times 12)$$

$$1{,}200 \leq \text{deposits in 12 months} \leq 1{,}800$$

Since the beginning dollars in bank account $= 700$, all possible values of s after 12 monthly deposits can be

$$1{,}200 + 700 \leq s \leq 1{,}800 + 700b \rightarrow$$

$$1{,}900 \leq s \leq 2{,}500$$

5. <u>43</u>

Determine the variables:

Let number of calls $= n$.

Determine the conditional relationship:

Monthly cost of n calls $= 0.16n$.

Monthly fixed fee + monthly cost of n calls must be less than or equal to $25.

$$17.99 + 0.16n \leq 25 \;\rightarrow\; 0.16n \leq 25 - 17.99 \;\rightarrow$$

$$0.16n \leq 7.01 \;\rightarrow\; n \leq 43.81 \rightarrow\; 43 \text{ maximum calls.}$$

6. <u>6</u>

Determine the variables:

Let number of candles $= c$.

Determine the conditional relationship:

Cost of 7 napkins $= 7 \times 2 = 14$.

Cost of c candles $= 1.5c$.

Cost of 7 napkins + cost of c candles must be less than or equal to $24.

$$14 + 1.5c \leq 24 \;\rightarrow\; 1.5c \leq 10 \;\rightarrow\; c \leq 6.6$$

The party planner can buy 6 whole candles.

7. <u>10</u>

Determine the conditional relationship:

Since maximum number of days is when they make the least number of bouquets in a day, use the lower hours they work per day.

Stu: least $=$ 8 bouquets in $=$ 1 day.

Stan: least $=$ 6 bouquets in $=$ 1 day.

Hence, $8 + 6 = 14$ bouquets per day.

Solve:

$$\frac{140}{14} = 10$$

Category 20 – Word Problems on Linear System of Inequalities

1. <u>B</u>

Identify the two variables:

Number of boxes weighing 20 pounds = m.

Number of boxes weighing 40 pounds = n.

Determine the conditional relationship:

Inequality 1: The number of boxes must be less than or equal to 55.

$$m + n \leq 55$$

This eliminates answer choices A and C.

Inequality 2: $m = 20$, and $n = 40$. Total weight of all the boxes must be less than or equal to 1,850.

$$20m + 40n \leq 1{,}850$$

This eliminates answer choice D.

2. <u>A</u>

Identify the two variables:

Number of exercise bikes = b.

Number of treadmills = t.

Determine the conditional relationship:

Inequality 1: The number of treadmills and bikes must be greater than or equal to 12.

$$b + t \geq 12$$

This eliminates answer choices B and D.

Inequality 2: Each $b = \$1{,}800$, and each $t = \$2{,}600$. Total cost must be less than or equal to $28,000.

$$1{,}800b + 2{,}600t \leq 28{,}000$$

This eliminates answer choice C.

3. <u>26</u>

Identify the two variables:

Let fiction books = c. Let non-fiction books = k.

Determine the conditional relationship:

Inequality 1: The total number of books must be greater than or equal to 50.

$$c + k \geq 50$$

Inequality 2: Each $c = \$12$, and each $k = \$15$. Maximum cost of all the books must be at the most $680.

$$12c + 15k \leq 680$$

Evaluate the system for k (non-fiction books):

$$c + k \geq 50 \ \rightarrow \ c \geq 50 - k$$

Hence, the minimum value of c is $50 - k$. Substitute $50 - k$ for c in $12c + 15k \leq 680$ to evaluate for the maximum value of k.

$$12(50 - k) + 15k \leq 680 \ \rightarrow$$

$$600 - 12k + 15k \leq 680 \ \rightarrow$$

$$3k \leq 680 - 600 \ \rightarrow \ 3k \leq 80 \ \rightarrow \ k \leq 26.6$$

Hence, the maximum value of k is 26 as books are a whole number.

4. <u>D</u>

Identify the two variables:

Number of on-line hours = l.

Number of in-home hours = h.

Determine the conditional relationship:

Inequality 1: The number of all hours per week must be less than or equal to 20.

$$l + h \leq 20$$

This eliminates answer choice C.

Inequality 2: Each $l = \$25$, and each $h = \$45$. Total from l and h must be greater than or equal to $600.

$$25l + 45h \geq 600$$

This eliminates answer choice A.

Inequality 3: h must be at least 10 but cannot be greater than 20.

$$10 \leq h \leq 20$$

This eliminates answer choice B.

Category 21 –Word Problems on Equal Variables in Linear Equations

1. <u>D</u>

Equate the two equations:

$$48 + \frac{1}{4}p = 150 - \frac{1}{2}p$$

Solve for p:

$$\frac{1}{4}p + \frac{1}{2}p = 150 - 48 \ \rightarrow \ \frac{3}{4}p = 102 \ \rightarrow \ p = 136$$

2. <u>C</u>

Equate the two equations:

$$10.50 + 4.50x = 36.50 + 2.50x$$

Solve for x:

$$4.50x - 2.50x = 36.50 - 10.50 \ \rightarrow \ x = 13$$

Solve c: Plug $x = 13$ into the equation.

$$c = 36.50 + (2.50 \times 13) = 36.50 + 32.50 = 69$$

3. A

Equate the two equations:

$$35 + 3p = 75 + p$$

Solve for p:

$$3p - p = 75 - 35 \rightarrow 2p = 40 \rightarrow p = 20$$

4. C

Determine the equation for manufacturing costs:

Let the manufacturing costs for next quarter $= C$.

$$C = 37{,}750 + 25W$$

Equate the two equations:

$$37{,}750 + 25W = 19{,}646 + 149W$$

Solve for W:

$$37{,}750 - 19{,}646 = 149W - 25W \rightarrow$$
$$18{,}104 = 124W \rightarrow W = 146$$

Section 4 – Drill

1. D

Determine the unknown number 1 (Samara): s miles per day for 4 days $= 4s$. This eliminates answer choices A and C.

Determine the unknown number 2 (Chuck): p miles per day for 6 days $= 6p$. This eliminates answer choice B.

2. A

Determine the components of the equation:

$21 =$ Average number of pages teacher will read each day.

$315 =$ Number of pages in the book.

$p =$ Number of pages left to read at the end of each day.

3. A

Determine the constant: Admission fee = \$40. This eliminates answer choices C and D.

Determine the unknown number: n rides for \$2 per ride $= 2n$. This eliminates answer choice B.

4. A

Identify the two variables:

Number of hours during morning shift $= M$.

Number of hours during night shift $= N$.

Determine the system of equations:

Equation 1: Total hours $= 10$.

$$M + N = 10$$

This eliminates answer choices B and C.

Equation 2: Each $M =$ \$18 and each $N =$ \$34. Total earnings = \$276.

$$18M + 34N = 276$$

This eliminates answer choice D.

5. B

Determine the two variables:

Number of packages shipped during off-peak hours $= x$.

Number of packages shipped during peak hours $= y$.

Determine the conditional relationship:

The total number of packages shipped per day at each location must be less than or equal to 150.

$$x + y \leq 150$$

This eliminates answer choices C and D.

The number of packages shipped during off-peak hours must be less than or equal to the number of packages shipped during peak hours.

$$x \leq y$$

This eliminates answer choice A.

6. D

Determine the variable:

Number of miles $= m$.

Determine the conditional relationship:

Miles driven must be less than or equal to the miles the car can go in 3.5 gallons of fuel.

In 3.5 gallons, the car can go $19.5 \times 3.5 = 68.25$ miles.

$$m \leq 68.25$$

7. B

$g(t)$ is a linear function. The input value of t determines the value of the function $g(t)$.

Hence, $g(12) = 19$ estimates that in 12 years from 2015, the tree is expected to be 19 feet tall.

8. C

Determine the unknown number 1 (walking on treadmill): t calories per minute for 35 minutes $= 35t$. This eliminates answer choices A and B.

Determine the unknown number 2 (running on treadmill): 6 calories per minute for 10 minutes $= 6 \times 10 = 60$. This eliminates answer choice D.

9. C

Determine the components of the equation:

$0.2 =$ Approximate increase in the Body Mass Index of an adult for each pound of weight. (This is same as "for each increase of 1 pound in weight, B increases by approximately 0.2".)

$B =$ Body Mass Index of an adult weighing p pounds.

10. C

Determine the two unknown variables:

$389c =$ total cost of c iPads.

$72d =$ total cost of d desks.

11. D

Determine the variables:

Pounds of cherry tomatoes = c.

Pounds of potatoes = p.

Determine the conditional relationship:

Total sold per day must be greater than 12 pounds.

$$c + p > 12$$

Total sold per week must be

$$c + p > (12 \times 7) \rightarrow \quad c + p > 84$$

12. 21

Solve: Plug the given values of $C = 3{,}965$ and $n = 3$ into the equation and solve for h.

$$3{,}965 = 500 + (55 \times 3h) \rightarrow 3{,}965 = 500 + 165h \rightarrow$$
$$165h = 3{,}965 - 500 \rightarrow 165h = 3{,}465 \rightarrow h = 21$$

13. D

Determine the components of the equation:

0.0012 = Approximate decrease in the barometric pressure, in inches Hg, at each foot above sea level.

29.92 = Barometric pressure, in inches Hg, at sea level.

B = Ending barometric pressure, in inches Hg, at a feet above sea level.

For the barometric pressure to decrease by 1, the value of $0.0012a$ must be 1. Hence, $a = \frac{1}{0.0012}$.

14. D

Determine the components of the equation:

68 = Average increase in the number of trout fish each month after January 2020 for m months.

p = Initial population in January 2020.

15. 20

Identify the two variables:

Let number of nickels = x.

Let number of dimes = y.

Determine the system of equations:

Equation 1: Total number of coins 50.

$$x + y = 50$$

Equation 2: Each nickel = \$0.05. Each dime = \$0.10.

$$0.05x + 0.1y = 3.50 \rightarrow$$
$$5x + 10y = 350$$

Solve the system for y (dimes):

$$\begin{array}{l} 5x + 10y = 350 \\ -5(x + y = 50) \end{array} \longrightarrow \begin{array}{l} \cancel{5x} + 10y = 350 \\ -\cancel{5x} - 5y = -250 \\ \hline 5y = 100 \\ y = 20 \end{array}$$

16. C

Determine the variable:

Number of teachers = g.

Determine the conditional relationship:

The number of teachers to be added per year is greater than or equal to 4 but less than or equal to 9. Hence,

$$4 \le g \text{ added per year} \le 9$$

In 3 years, all possible values of teachers added are

$$4 \times 3 \le g \text{ added in 3 years } \le 9 \times 3$$
$$12 \le g \text{ added in 3 years} \le 27$$

Since currently there are 52 teachers, all possible values of the total number of teachers in 3 years can be

$$12 + 52 \ \le g \le 27 + 52$$
$$64 \ \le g \le 79$$

17. 56

Identify the two variables:

Let 6-pack container = x.

Let 14-pack container = y.

Set up the system of equations:

Equation 1: Total number of containers = 12.

$$x + y = 12$$

Equation 2: Number of cupcakes in 6-pack containers = $6x$ and number of cupcakes in 14-pack containers = $14y$. Total number of cupcakes = 104.

$$6x + 14y = 104$$

Solve the system for y (14-pack containers):

$$\begin{array}{l} 6x + 14y = 104 \\ -6(x + y = 12) \end{array} \longrightarrow \begin{array}{l} \cancel{6x} + 14y = 104 \\ -\cancel{6x} - 6y = -72 \\ \hline 8y = 32 \rightarrow y = 4 \end{array}$$

The number of cupcakes in 14-pack containers $= 14 \times 4 = 56$.

18. 8

Form the equations:

Let the number of extra-large burgers = b.

Equation for cost is

$$41.50 + 0.45b$$

Equation for profit is

$$29.50 + 1.95b$$

Set the equations equal:

$$41.50 + 0.45b = 29.50 + 1.95b$$

Solve for b:

$$1.95b - 0.45b = 41.50 - 29.50 \rightarrow$$
$$1.5b = 12 \rightarrow b = \frac{12}{1.5} = 8$$

19. D

Identify the two variables:

Number of hours per week teaching Algebra I = m.

Number of hours per week teaching Spanish = n.

Determine the conditional relationship:

Equation 1: Total number of hours per week must be less than or equal to 15.

$$m + n \leq 15$$

This eliminates answer choice A.

Equation 2: Each $m = \$50$ and each $n = \$38$. The earnings from m and n must be greater than or equal to \$500 per week.

$$50m + 38n \geq 500$$

This eliminates answer choice B.

Equation 3: n must be at least 7 per week. Since the maximum number of hours per week is 15, n is between 7 and 15 hours.

$$7 \leq n \leq 15$$

This eliminates answer choice C.

20. B

Identify the two variables:

Let number of tickets sold at reduced price = x.

Let number of tickets sold at regular price = y.

Determine the system of equations:

Equation 1: Total number of tickets sold = 240.

$$x + y = 240$$

Equation 2: Price of each $x = \$9$ and price of each $y = \$29$. Total price of all tickets sold = \$4,960.

$$9x + 29y = 4{,}960$$

Solve the system for y (regular price):

$$\begin{array}{l} 9x + 29y = 4{,}960 \\ -9(x + y = 240) \end{array} \longrightarrow \begin{array}{r} \cancel{9x} + 29y = 4{,}960 \\ -\cancel{9x} - 9y = -2{,}160 \\ \hline 20y = 2{,}800 \rightarrow y = 140 \end{array}$$

21. 22

At present: Let shelves = x, and books per shelf = y. Hence, $y = \frac{154}{x}$.

After addition: $y - 8 = \frac{154}{x+4} \rightarrow y = \frac{154}{x+4} + 8$.

Equate and solve.

$$\frac{154}{x} = \frac{154}{x+4} + 8 \rightarrow \frac{154}{x} - \frac{154}{x+4} = 8 \rightarrow$$

$$\frac{154(x+4)}{x(x+4)} - \frac{154(x)}{x(x+4)} = 8 \rightarrow$$

$$\frac{154x + 616 - 154x}{x(x+4)} = 8 \rightarrow$$

$$616 = 8x(x+4) \rightarrow 616 = 8x^2 + 32x \rightarrow$$

$$8x^2 + 32x - 616 = 0 \rightarrow x^2 + 4x - 77 = 0 \rightarrow$$

$$(x+11)(x-7) = 0 \rightarrow x = -11 \text{ and } 7.$$

Hence, $x = 7$, and books per shelf $y = \frac{154}{x} = \frac{154}{7} = 22$.

22. C

Determine the unknown number:

Students enrolled in 2015-16 = 176.

Students enrolled in 2020-21 = 236.

Total increase in 5 years = $236 - 176 = 60$. Hence, average increase per year = $\frac{60}{5} = 12$.

Average increase in t years = $12t$.

Determine the equation: Number of students r in t years after 2020-21 = students enrolled in 2020-21 + average increase per year for t years.

$$r = 236 + 12t$$

23. 50

Form the equations:

Let the number of bags filled with cookies = c.

<u>Equation for 5 cookies per bag</u>

Each bag contains 5 cookies. Hence, the number of cookies in c bags = $5c$.

4 additional bags are required for the remaining cookies. Hence, the number of cookies for which 4 extra bags are required = $5 \times 4 = 20$.

$$\text{Total cookies} = 5c + 20$$

<u>Equation for 10 cookies per bag</u>

Each bag contains 10 cookies. Hence, the number of cookies in c bags = $10c$.

1 bag is extra after filling the cookies in bags. Hence, there are $10 \times 1 = 10$ cookies less for c bags.

$$\text{Total cookies} = 10c - 10$$

Equate and solve for c: In both the equations, the total number of cookies is same.

$$5c + 20 = 10c - 10 \rightarrow$$

$$10c - 5c = 20 + 10 \rightarrow c = 6$$

Solve for the number of cookies: Plug $c = 6$ into any equation.

$$5c + 20 \rightarrow (5 \times 6) + 20 = 30 + 20 = 50$$

24. 83

Identify the two variables:

Let number of rice bags= x. Let number of flour bags= y.

Determine the conditional relationship:

Inequality 1: The total number of bags of rice and flour must be greater than or equal to 150.

$$x + y \geq 150$$

Inequality 2: $x = \$10$. $y = \$7$. Maximum = \$1,300.

$$10x + 7y \leq 1{,}300$$

Evaluate the system for x (number of rice bags):

$$x + y \geq 150 \rightarrow y \geq 150 - x$$

The minimum value of y is $150 - x$. Substitute $150 - x$ for y in $10x + 7y \leq 1{,}300$ and evaluate x.

$$10x + 7(150 - x) \leq 1{,}300 \rightarrow$$

$$10x + 1{,}050 - 7x \leq 1{,}300 \rightarrow$$

$$3x \leq 1{,}300 - 1{,}050 \rightarrow 3x \leq 250 \rightarrow x \leq 83.3$$

Hence, the maximum value of x is 83.

Section 5 – Polynomial and Undefined Functions

Category 22 – Standard Form Polynomial Functions

1. <u>72</u>

Plug the given value of x into the function:

$$f(3) = (3)^4 - (3)^3 + 18 = 81 - 27 + 18 = 72$$

2. <u>D</u>

Plug in the given value of x in the function:

$$f(-2) = (-2)^4 - 3(-2)^2 - 4 = 16 - 12 - 4 = 0$$

$$f(1) = (1)^4 - 3(1)^2 - 4 = 1 - 3 - 4 = -6$$

Solve for $f(-2) - f(1)$:

$$0 - (-6) = 6$$

3. <u>B</u>

Solve for b: Divide both sides by 3.

$$\frac{3f(b)}{3} = \frac{21}{3} \rightarrow f(b) = 7$$

$f(x) = 4x^4 - 57$ can be written as $f(b) = 4b^4 - 57$.

Determine the value of b when $f(b) = 7$.

$$7 = 4b^4 - 57 \rightarrow 4b^4 = 57 + 7 = 64 \rightarrow$$

$$b^4 = 16 \rightarrow b^4 = 2^4 \rightarrow b = \pm 2 \rightarrow b = 2$$

4. <u>C</u>

Plug the given value of x into the function:

$$f(2x+5) = \frac{3(2x+5) - 7}{2} = \frac{6x + 15 - 7}{2} =$$

$$\frac{6x+8}{2} = 3x + 4$$

5. <u>B</u>

Plug the given value of x into $f(x)$:

$$f(-3) = (-3)^5 + 4(-3)^4 + (-3) =$$

$$-243 + 4(81) - 3 = -243 + 324 - 3 = 78$$

Plug the given value of x into $g(x)$:

$$g(5) = 4(5)^3 - 17(5)^2 - 74 = 500 - 425 - 74 = 1$$

Determine $f(-3) \times g(5) = 78 \times 1 = 78$

6. <u>B</u>

Plug the given value of x into the function:

$$g(10): \frac{10^2 m + n}{2} = 300 \rightarrow \frac{100m + n}{2} = 300 \rightarrow$$

$$100m + n = 2 \times 300 \rightarrow 100m + n = 600$$

$$g(20): \frac{20^2 m + n}{2} = 1{,}500 \rightarrow \frac{400m + n}{2} = 1{,}500 \rightarrow$$

$$400m + n = 2 \times 1{,}500 \rightarrow 400m + n = 3{,}000$$

Solve for m: Create a system of equations and solve.

$$\begin{array}{c} 400m + n = 3{,}000 \\ -(100m + n = 600) \end{array} \longrightarrow \begin{array}{c} 400m + n = 3{,}000 \\ -100m - n = -600 \\ \hline 300m = 2{,}400 \rightarrow m = 8 \end{array}$$

7. <u>D</u>

Solve for t: Add the two equations and equate them to 0.

$$(3t^4 - 2t - 4) + (-3t^4 + 5t - 5) = 0 \rightarrow$$

$$3t - 9 = 0 \rightarrow 3t = 9 \rightarrow t = 3$$

Category 23 – Tables and Graphs of Polynomial Functions

1. <u>55</u>

At the y-intercept, $x = 0$. Hence, when $x = 0$ is plugged into $f(x) = 3x^4 - 4x^3 + 55$, the result is 55.

2. <u>1</u>

Read the values of $f(x)$ from the table:

$f(1) = -3$. $f(2) = 1$. $f(4) = 4$.

Solve for $\frac{f(2) - f(1)}{f(4)}$: Plug in the above values.

$$\frac{1 - (-3)}{4} = \frac{4}{4} = 1$$

3. <u>A</u>

Read the values of x from the graph for $y = 3$:

$$x = -2 \text{ and } 2$$

Hence, $f(-2)$ and $f(2)$ satisfy $y = f(x) = 3$.

4. <u>B</u>

4 solutions of c implies that 4 values of x define $y = c$.

Count the number of x values for each value of y given in the answer choices:

Answer choice A: For $y = -3$, there are 2 values of x.

Answer choice B: For $y = -2$, there are 4 values of x.

Answer choice C: For $y = 0$, there are 2 values of x.

Answer choice D: For $y = 1$, there are 2 values of x.

5. <u>D</u>

Read the value of $g(x)$ from the table: Since $x = 6$ is not in the table, determine a value of x that can give $g(3x) = g(6)$ and can be read from the table.

If x is given the value of 2, then

$$2g(2) = g(3 \times 2) \rightarrow 2g(2) = g(6)$$

From the table, $g(2) = 5$. Hence,

$$2g(2) = g(6) \rightarrow 2 \times 5 = g(6) \rightarrow g(6) = 10$$

Category 24 – Nested Polynomial Functions

1. <u>8</u>

Since $f(3) = 5$, $g(f(3)) = g(5)$.

Plug $x = 5$ into the outside function $g(x)$:

$$g(5) = \frac{(2 \times 5^3) - 98}{19} = \frac{(2 \times 125) - 98}{19} = \frac{152}{19} = 8$$

2. <u>C</u>

Plug the given value of x into all the functions:

$$g(7) = 3f(7-1) - 3 = 3f(6) - 3 =$$
$$3\big((4 \times 6) - 19\big) - 3 = 3(24 - 19) - 3 =$$
$$3(5) - 3 = 15 - 3 = 12$$

3. <u>D</u>

Plug the given value of x into the nested function $g(x)$:

$$g(3) = 3^3 - 20 = 27 - 20 = 7$$

Plug in $x = 7$ in the outside function $f(x)$:

$$f\big(g(3)\big) = f(7) = 7^2 - 7 = 49 - 7 = 42$$

4. <u>B</u>

Plug the given value of x into $g(x)$ and solve:

Set $h(x) = -5$.

$$-5 = c - g(2) \ \rightarrow \ -5 = c - (2^2 + 4) \ \rightarrow$$
$$-5 = c - 8 \ \rightarrow \ c = -5 + 8 = 3$$

5. <u>C</u>

Read the y value from the graph for the inside function:

$$h(-3) = 1$$

Solve the equation for function g:

$$g\big(h(-3)\big) = g(1) = 2(1)^4 + 3 = 5$$

Solve for $g\big(h(-3)\big) - 2$: $5 - 2 = 3$

6. <u>A</u>

$g(f(3))$:

Read the $f(x)$ value for the inside function:

$$f(3) = 0$$

Read the $g(x)$ value for the outside function:

$$g\big(f(3)\big) = g(0) = 5$$

$f(g(-1))$:

Read the $g(x)$ value for the inside function:

$$g(-1) = 7$$

Read the $f(x)$ value for the outside function:

$$f\big(g(-1)\big) = f(7) = 4$$

Solve the equation:

$$g\big(f(3)\big) + f\big(g(-1)\big) + k = 1 \ \rightarrow \ 5 + 4 + k = 1 \ \rightarrow$$
$$9 + k = 1 \ \rightarrow \ k = -8$$

Category 25 – Zeros, Factors, and Factored Form Polynomial Functions

1. <u>C</u>

Determine the points on the x-axis where the graph crosses: The graph crosses the x-axis at 1 and 3. The corresponding factors are $(x-1)$ and $(x-3)$.

Determine the points on the x-axis where the graph touches the x-axis and curves back: The graph touches the x-axis at -2 and curves back. The corresponding factors are $(x+2)^2$.

The factors are $(x-1)(x-3)(x+2)^2$.

2. <u>B</u>

Determine the zeros:

For x, the zero is 0.

For $(x-1)$, the zero is 1.

For $(x-3)$, the zero is 3.

For $(x+2)^2$, the distinct zero is -2.

The distinct zeros in ascending order are -2, 0, 1, and 3.

3. <u>100</u>

Determine the values of x where $y = 0$:

The values of x where $y = h(x) = 0$ are 2, 5 and 10.

Determine the product of the zeros:

$$2 \times 5 \times 10 = 100$$

4. <u>C</u>

Determine the zeros:

For $(x-4)$, the zero is 4.

For $(x^2 - 4)$, the two factors are $(x+2)(x-2)$. The two zeros are -2 and 2.

There are three distinct zeros.

5. <u>D</u>

Determine the values of x where $y = 0$.

The two values of x where $y = f(x) = 0$ are -2 and -8.

Hence, the two factors are $(x+2)(x+8)$.

Category 26 – Graph Transformations of Polynomial Functions

1. <u>C</u>

Determine the translated graph: Add 2 to x.

$$y = f(x+2) = (x+2+2)(x-1+2)(x-4+2)$$
$$= (x+4)(x+1)(x-2)$$

Determine which table contains all the points on the graph of $f(x+2)$: Plug in the x-value of each point in a table and match to corresponding y-value.

All the points in answer choice C are on the graph of $y = f(x+2)$.

2. <u>B</u>

Determine the translation: Subtract 7 from each value of x.

$$f(x) = (2(x-7)-3)(x-7+7)(x-7+8)^2 =$$
$$(2x-17)(x)(x+1)^2 = x(2x-17)(x+1)^2$$

3. <u>28</u>

Determine the translation:

f is 12 units down of g. Hence,

$$f(x) = g(x) - 12$$

Plug in $x = 0$:

$f(0) = g(0) - 12 = (0-5)(0-1)(0+8) - 12 = 28$

Category 27 – Remainder in Polynomial Functions

1. <u>D</u>

Factor Theorem:

Since $p(-\frac{1}{2}) = 0$, $x + \frac{1}{2}$ is a factor of $p(x)$.

$$x + \frac{1}{2} \rightarrow 2x + 1$$

2. <u>B</u>

Plug the value of x from each answer choice into the equation and evaluate for $p(x) = 0$ (Factor Theorem):

Start with answer choice B.

$$3x - 1 \rightarrow x = \frac{1}{3}$$

$$p\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right)^3 - \left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 2 = 0 \rightarrow$$
$$3\left(\frac{1}{27}\right) - \frac{1}{9} - 2 + 2 = 0 \rightarrow$$
$$\frac{1}{9} - \frac{1}{9} - 2 + 2 = 0$$

Since $p\left(\frac{1}{3}\right) = 0$, $3x - 1$ is a factor of $p(x)$.

(Note that this equation can also be factored. However, factoring may not always be easy in polynomials.

$$3x^3 - x^2 - 6x + 2 = x^2(3x-1) - 2(3x-1)$$

The two factors are $(x^2 - 2)$ and $(3x - 1)$.)

3. <u>D</u>

Factor Theorem:

When $p(x)$ is divided by $x + 5$, the remainder is 4. Hence, $x + 5$ is not a factor of $p(x)$. Answer choice D is correct.

4. <u>2</u>

Plug $x = -3$ into the equation (Remainder Theorem):

$$f(-3) = 3(-3)^3 + 8(-3)^2 + 11 =$$
$$(3 \times -27) + (8 \times 9) + 11 = -81 + 72 + 11 = 2$$

5. <u>B</u>

Plug $x = 2$ into the equation and set equation to 0 (Factor Theorem):

$$p(2) = 3(2)^3 - 3(2)^2 - 2c + 2 = 0 \rightarrow$$
$$(3 \times 8) - (3 \times 4) - 2c + 2 = 0 \rightarrow$$
$$24 - 12 - 2c + 2 = 0 \rightarrow$$
$$14 - 2c = 0 \rightarrow 2c = 14 \rightarrow c = 7$$

6. <u>C</u>

Plug $x = -4$ into the expression (Remainder Theorem):

A is the remainder when $2x^2 + 9x + 5$ is divided by $x + 4$. Hence,

$$A = 2(-4)^2 + 9(-4) + 5 =$$
$$(2)(16) + (9)(-4) + 5 = 32 - 36 + 5 = 1$$

7. <u>A</u>

The factor of the polynomial $p(x)$ will result in $p(x) = 0$ (Factor Theorem).

$x = 1$ plugged into the equation should result in 0.

Evaluate each answer choice: Plug $x = 1$ into the equations given in answer choices.

By looking at the equations of answer choices B and C it can be observed that for $x = 1$, the equations will not equal to 0. They can be eliminated. Answer choice A results in 0. See below.

$$p(1) = -2(1)^2 - 3(1) + 5 = -2 - 3 + 5 = 0$$

Hence, $x - 1$ is a factor of the polynomial $p(x) = -2x^2 - 3x + 5$.

Category 28 – Undefined Functions

1. A

Set denominator = 0:

$$6x + 2x + 16 = 0$$

Solve:

$$8x + 16 = 0 \rightarrow 8x = -16 \rightarrow x = -2$$

2. C

Set denominator = 0:

$$3x - 12 = 0$$

Solve:

$$3x = 12 \rightarrow x = 4$$

3. D

Set denominator = 0:

$$(x^2 - 11x + 27) + (x - 2) = 0$$

Solve:

$$x^2 - 10x + 25 = 0 \rightarrow (x - 5)^2 \rightarrow x = 5$$

4. D

Set denominator = 0:

$$(x + 4)^2 - (16x + 1) = 0$$

Solve: Simplify and factor.

$$x^2 + 8x + 16 - 16x - 1 = 0 \rightarrow x^2 - 8x + 15 \rightarrow$$
$$(x - 3)(x - 5) \rightarrow x = 3 \text{ and } 5$$

Section 5 – Drill

1. A

Determine the points on the x-axis where the graph crosses: The graph crosses the x-axis at 1. The corresponding factor is $(x - 1)$.

Determine the points on the x-axis where the graph touches the x-axis and curves back: The graph touches the x-axis and curves back at -1 and 3. The corresponding factors are $(x + 1)^2$ and $(x - 3)^2$.

The factors are $(x - 1)(x - 3)^2(x + 1)^2$.

2. 60

Plug the given value of x into the function:

$$f(-3) = (-3)^4 - 21 = 81 - 21 = 60$$

3. 116

Plug the given value of x into the function:

$$g(24) = \big((0.2 \times 24) + 1\big)(24 - 4) =$$
$$(4.8 + 1)(20) = 5.8 \times 20 = 116$$

4. 9

Plug $x = 3$ into the equation (Remainder Theorem):

$$p(3) = 3^3 - (7 \times 3) + 3 = 27 - 21 + 3 = 9$$

5. D

Determine the x-intercept: Set $y = g(x) = 0$.

$$0 = 3x - 6 \rightarrow 3x = 6 \rightarrow x = 2$$

Hence, the x-intercept is (2, 0). This can also be written as (2, $g(2)$), since $y = g(2) = 0$.

6. B

Determine from the table the values of x where $y = 0$:

$$x = 2$$

Hence, the factor is $x - 2$.

7. C

Determine the zeros:

For $2x$, the zero is 0.

For $(x - k)^2$, the distinct zero is k.

For $(x + k)^2$, the distinct zero is $-k$.

There are three distinct zeros.

8. 16

Solve for a: Plug $x = 2$ into the equation and set it to 10.

$$h(2) = a(2^2) + a(2) + 4 = 10 \rightarrow$$
$$4a + 2a + 4 = 10 \rightarrow 6a = 6 \rightarrow a = 1$$

Plug the given value of x into the function and $a = 1$:

$$h(3) = 1(3^2) + 1(3) + 4 \rightarrow 9 + 3 + 4 = 16$$

9. A

Read the value of y from the graph for $x = -1$:

$$y = t(-1) = -2$$

Hence, $g(m) = g(-1) = -2$.

Count the number of x values from the graph for $y = -2$: Draw a line through $y = -2$ and count.

There are 3 distinct values of x that define $g(m) = -2$.

10. B

Determine the x-intercepts:

For $(x - 1)$, the x-intercept of the x-intercept is 1.

For $(2x - 3)$, the x-intercept of the x-intercept is $\frac{3}{2}$.

For $(3x + 2)$, the x-intercept of the x-intercept is $-\frac{2}{3}$.

This eliminates option II.

11. D

Factor Theorem:

Since $p(-\frac{2}{3}) = 0$, $x + \frac{2}{3}$ is a factor of $p(x)$.

$$x + \frac{2}{3} \rightarrow 3x + 2$$

12. B

Evaluate each answer choice: Plug the answer choices into the given equation and evaluate for $p(x) = 0$ (Factor Theorem).

Start with answer choice B.

$$p(-2) = 5(-2)^3 + 8(-2)^2 - 3(-2) + 2 =$$
$$(5 \times -8) + (8 \times 4) - (3 \times -2) + 2 =$$
$$-40 + 32 + 6 + 2 = 0$$

Since $p(-2) = 0$, -2 is a zero of $p(x)$.

13. A

Read the value of y from the graph for $x = 2$:

$$h(2) = 4$$

Hence, $h(t) = h(2) = 4 \rightarrow h(t) = 4$

Read the values of x from the graph for $y = 4$:

There are two values of x for $y = h(t) = 4$.

$$x = -1 \text{ and } 2$$

Answer choice A is -1.

14. 35

Plug the given value of x into all the functions:

$$g(8) = 2f(8 + 2) - 5 = 2f(10) - 5 =$$
$$2\big((3 \times 10) - 10\big) - 5 = 2(30 - 10) - 5 =$$
$$2(20) - 5 = 40 - 5 = 35$$

15. C

For $g(x)$, the x-intercepts are 3, 8, and -1.

A translation of 3 units left will result in $3 - 3 = 0$, $8 - 3 = 5$, and $-1 - 3 = -4$. Hence,

$$a + b + c = 0 + 5 - 4 = 1$$

16. 20

Since $g(b) = 10$, $f(g(b)) = f(10)$.

Plug $x = 10$ into the outside function f:

$$f(10) = \frac{(17.8 \times 10^2) + b}{3} = \frac{1{,}780 + b}{3}$$

Determine the value of b: It is given $f(10) = 600$.

$$\frac{1{,}780 + b}{3} = 600 \rightarrow 1{,}780 + b = 1{,}800 \rightarrow b = 20$$

17. 4

Plug the given value of x into the function:

$$2f(3) = f(3 + 2) \rightarrow 2f(3) = f(5)$$

It is given that $f(5) = 8$. Hence,

$$2f(3) = 8$$

Divide both sides by 2.

$$\frac{2f(3)}{2} = \frac{8}{2} \rightarrow f(3) = 4$$

18. 3

Solve for x: Plug the definitions of $f(x)$ and $h(x)$ into the given equation:

$$f(x) - h(x) = 9 \rightarrow (9x - 1) - (5x + 2) = 9 \rightarrow$$
$$9x - 1 - 5x - 2 = 9 \rightarrow 4x = 12 \rightarrow x = 3$$

19. A

$f(g(4))$:

Read the g(x) value for the inside function:

$$g(4) = -2$$

Read the $f(x)$ value for the outside function:

$$f\big(g(4)\big) = f(-2) = 9$$

$g(f(4))$:

Read the $f(x)$ value for the inside function:

$$f(4) = 1$$

Read the $g(x)$ value for the outside function:

$$g\big(f(4)\big) = g(1) = 5$$

Solve the equation:

$$k = f\big(g(4)\big) - g\big(f(4)\big) = 9 - 5 = 4$$

20. C

Set denominator $= 0$ and solve:

$$(x^2 - x - 9) + 3(x - 2) = 0 \rightarrow$$
$$x^2 - x - 9 + 3x - 6 = 0 \rightarrow x^2 + 2x - 15 = 0 \rightarrow$$
$$(x + 5)(x - 3) = 0 \rightarrow x = -5 \text{ and } 3$$

21. C

Plug the given value of x into the function: $x = 5 - c$ and set it to 0.

$$2(5 - c)(5 - c - 8)(5 - c + 9)^2 = 0 \rightarrow$$
$$(5 - c)(-3 - c)(14 - c)^2 = 0$$

Hence, the values of x are

$$5 - c = 0 \rightarrow c = 5$$
$$-3 - c = 0 \rightarrow c = -3$$
$$14 - c = 0 \rightarrow c = 14$$
$$\text{sum} = 5 - 3 + 14 = 16$$

Section 6 – Quadratic Equations and Parabola

Category 29 – Quadratic Equations and Factors

1. A

Determine the roots: Since x^2 is the only x term in this example, isolate it on one side of the equation.

$$x^2 - 21 = 175 \rightarrow x^2 = 21 + 175 = 196$$

196 is a perfect square of 14. Hence,

$$x^2 = 196 = 14^2 \rightarrow x \pm 14$$

2. A

Determine the roots: Remove multiple of 2 to simplify.

$$2(x^2 + 3x - 40) = 0 \rightarrow x^2 + 3x - 40 = 0$$

The two multiples of -40 that add to 3 are -5 and 8. Hence,

$$(x - 5)(x + 8) = 0$$
$$x = 5 \text{ and } -8$$

-8 is one of the answer choices.

3. B

Determine the roots:

$$x^2 - 7x + 12 = 0$$

The two multiples of 12 that add to -7 are -3 and -4. Hence,

$$(x - 3)(x - 4) = 0$$
$$x = 3 \text{ and } 4$$

4. C

Determine the factors: Since $a > 0$, the factoring below is shown using the formula. Note that this equation can be factored without the formula depending on the student's comfort level.

$a = 2.\ b = -5.\ c = -12.$

$$\frac{-(-5) \pm \sqrt{(-5)^2 - (4 \times 2 \times -12)}}{2 \times 2} = \frac{5 \pm \sqrt{25 + 96}}{4} =$$
$$\frac{5 \pm \sqrt{121}}{4} = \frac{5 \pm 11}{4}$$

The two roots are

$$\frac{5 + 11}{4} = \frac{16}{4} = 4 \text{ and } \frac{5 - 11}{4} = -\frac{6}{4} = -\frac{3}{2}$$

The two factors are

$$(x - 4)\left(x + \frac{3}{2}\right) \rightarrow (x - 4)(2x + 3)$$

5. D

Form a quadratic equation:
Remove parentheses and simplify.

$$3x^2 + 12x + 1 = 2x^2 + 10x + 6 \rightarrow$$
$$3x^2 + 12x + 1 - 2x^2 - 10x - 6 = 0 \rightarrow$$
$$x^2 + 2x - 5 = 0$$

Determine the roots: Since the answer choices have square roots, use the quadratic formula.

$a = 1.\ b = 2.\ c = -5.$

$$\frac{-2 \pm \sqrt{2^2 - (4 \times 1 \times -5)}}{2 \times 1} = \frac{-2 \pm \sqrt{4 + 20}}{2} =$$
$$\frac{-2 \pm \sqrt{24}}{2}$$

The two multiples of $\sqrt{24}$ are $\sqrt{6 \times 4}$ where $\sqrt{4} = 2$.

$$\frac{-2 \pm \sqrt{4 \times 6}}{2} = \frac{-2 \pm 2\sqrt{6}}{2} = \frac{-2}{2} \pm \frac{2\sqrt{6}}{2} = -1 \pm \sqrt{6}$$

The two solutions are $-1 - \sqrt{6}$ and $-1 + \sqrt{6}$. Answer choice D is one of them.

6. D

Determine the factors:

$a = 1.\ b = -14.\ c = 26.$

$$\frac{-(-14) \pm \sqrt{(-14)^2 - (4 \times 1 \times 26)}}{2} =$$
$$\frac{14 \pm \sqrt{196 - 104}}{2} = \frac{14 \pm \sqrt{92}}{2} = \frac{14 \pm \sqrt{4 \times 23}}{2} =$$
$$\frac{14 \pm 2\sqrt{23}}{2} = 7 \pm \sqrt{23}$$

Hence, $7 \pm \sqrt{m} = 7 \pm \sqrt{23}$

$$m = 23$$

7. D

Determine the roots: Rewrite as quadratic equation.
Let $n = \sqrt{x}$. Hence, $n^2 = (\sqrt{x})(\sqrt{x}) = x$.
The equation becomes

$$x - 31\sqrt{x} + 108 = 0 \rightarrow n^2 - 31n + 108 = 0$$

The two multiples of 108 that add to -31 are -4 and -27. Hence,

$$(n - 4)(n - 27) = 0 \rightarrow n = 4 \text{ and } 27$$

Substitute $\sqrt{x}$ for n and determine the two values of x.

$$\sqrt{x} = 4 \rightarrow x = 4^2 = 16$$
$$\sqrt{x} = 27 \rightarrow x = 27^2 = 729$$

729 is one of the answer choices.

Category 30 – Quadratic Equations and Number of Roots

1. A

Determine number of solutions using the discriminant:

$a = 3.\ b = 4.\ c = 7.$

$$b^2 - 4ac = (4)^2 - (4 \times 3 \times 7) = 16 - 84 = -68$$

Since discriminant < 0, there is no real solution.

2. C

Determine number of solutions using the discriminant:

$a = 7.\ b = -10.\ c = 4.$

$$b^2 - 4ac = (-10)^2 - (4 \times 7 \times 4) =$$
$$100 - 112 = -12$$

Since discriminant < 0, there is no real solution.

3. 2

Set the discriminant to 0:

$a = a.\ b = -8.\ c = 8.$

Since the equation has one solution, the discriminant is 0.

$$b^2 - 4ac = 0 \ \rightarrow\ (-8)^2 - (4 \times a \times 8) = 0 \ \rightarrow$$
$$64 - 32a = 0 \ \rightarrow\ a = 2$$

4. A

Set the discriminant to < 0:

Rearrange before determining the discriminant.

$$4x^2 - 4x = k \ \rightarrow\ 4x^2 - 4x - k = 0$$

$a = 4.\ b = -4.\ c = -k.$

Since the equation has no real solution, the discriminant must be less than 0.

$$b^2 - 4ac < 0 \ \rightarrow\ (-4)^2 - (4 \times 4 \times -k) < 0 \ \rightarrow$$
$$16 + 16k < 0 \ \rightarrow\ 16k < -16 \ \rightarrow\ k < -1$$

Only answer choice A is less than -1.

5. D

Set the discriminant to 0:

$a = a.\ b = 6.\ c = c.$

Since the equation has one solution, the discriminant is 0.

$$b^2 - 4ac = 0 \ \rightarrow\ (6)^2 - (4 \times a \times c) = 0 \ \rightarrow$$
$$36 - 4ac = 0 \ \rightarrow\ 4ac = 36 \ \rightarrow\ ac = 9$$

6. C

Set up the discriminant for each answer choice: Since the equation has one solution, the correct answer choice will have discriminant $= 0$.

Only answer choice C has discriminant $= 0$. See below.

$$b^2 - 4ac = (-6)^2 - (4 \times 9 \times 1) = 36 - 36 = 0$$

7. 11

Set the discriminant to < 0:

$a = 1.\ b = b.\ c = 36.$

Since the equation has no real solution, the discriminant must be less than 0.

$$b^2 - 4ac < 0 \ \rightarrow\ (b)^2 - (4 \times 1 \times 36) < 0 \ \rightarrow$$
$$b^2 - 144 < 0 \ \rightarrow\ b^2 < 144 \ \rightarrow\ b < \pm 12$$

Since b is positive, $b < 12$. Hence, the greatest possible integer value of b is 11.

Category 31 — Sum and Product of Quadratic Roots

1. B

Use the product formula: All the values of n is referring to the roots of the quadratic equation.

$a = 2.\ c = -20.$

$$\frac{c}{a} = \frac{-20}{2} = -10$$

2. 5

Use the sum formula:

$a = 3.\ b = -15.$

$$\text{Sum of roots} = -\frac{b}{a} = -\frac{-15}{3} = 5$$

3. D

Use the sum and product formulas:

Convert to the standard form before using the formulas.

$$3x^2 + 19x = 6x - 5 \ \rightarrow\ 3x^2 + 19x - 6x + 5 = 0 \rightarrow$$
$$3x^2 + 13x + 5 = 0$$

$a = 3.\ b = 13.\ c = 5.$

$$m = \text{product of roots} = \frac{c}{a} = \frac{5}{3}$$
$$n = \text{sum of roots} = -\frac{b}{a} = -\frac{13}{3}$$

Solve for product of roots $-$ sum of roots:

$$m - n = \frac{5}{3} - \left(-\frac{13}{3}\right) = \frac{5}{3} + \frac{13}{3} = \frac{18}{3} = 6$$

4. C

Use the product formula: Since the value of b is not given, use the product formula.

$a = 2.\ \ c = -12.$

$$\frac{c}{a} = \frac{-12}{2} = -6$$

Evaluate each answer choice for the product of the two given zeros. The correct product = -6.

Answer choice A = $-2 \times -12 = 24$. This is incorrect.

Answer choice B = $-2 \times -6 = 12$. This is incorrect.

Answer choice C = $-1 \times 6 = -6$. This is correct.

Answer choice D = $1 \times 6 = 6$. This is incorrect.

5. A

Use the sum formula: Since the value of c is not given, use the sum formula.

$a = 2.\ \ b = -7.$

$$\text{sum of roots using formula} = -\frac{b}{a} = -\frac{-7}{2} = \frac{7}{2}$$

Determine the second root: It is given that one of the roots = 2.

Let the second root = x. Hence,

$$\text{sum of roots by adding} = 2 + x$$

Equate the two sums of the roots.

$$2 + x = \frac{7}{2} \rightarrow x = \frac{7}{2} - 2 = \frac{7-4}{2} = \frac{3}{2}$$

6. C

Use the product formula: Since the value of b is not given, use the product formula.

$a = 2.\ \ c = 9.$

$$\text{Product of roots using formula} = \frac{c}{a} = \frac{9}{2}$$

Determine the second root:

It is given that one of the roots = 3.

Let the second root = x. Hence,

$$\text{product of roots by multiplying} = 3 \times x$$

Equate the two products of the roots.

$$3 \times x = \frac{9}{2} \rightarrow x = \frac{9}{2} \times \frac{1}{3} = \frac{3}{2}$$

Category 32 – Standard Form Equation of a Parabola

1. A

Determine the x-coordinate of the vertex: It is the midpoint of the two x-intercepts.

The midpoint of -5 and 11 is 3. Use the midpoint formula if unsure.

2. B

Determine the x-coordinate of the vertex:

$a = 1.\ \ b = -8.$

$$-\frac{b}{2a} = -\frac{-8}{2 \times 1} = 4$$

This eliminates answer choices A and D.

Determine the y-coordinate of the vertex: Plug $x = 4$ into the equation.

$$4^2 - (8 \times 4) + 17 = 16 - 32 + 17 = 1$$

This eliminates answer choice C.

3. C

Determine y-coordinate of the vertex:

p = the maximum daily profit is the y-coordinate of the vertex. From the graph, the y-coordinate of the vertex is 250.

4. A

Determine if the parabola opens upward or downward:

Since a is negative, the parabola opens downward. This eliminates answer choices B and D.

Determine the x-coordinate of the vertex: Since the remaining answer choices have the same y-intercept, determine the x-coordinate of the vertex for further elimination.

$a = -1.\ \ b = -2.$

$$-\frac{b}{2a} = -\frac{-2}{2 \times -1} = -1$$

This eliminates answer choice C.

5. C

Determine if the parabola opens upward or downward: Since the parabola opens upward, a is positive. This eliminates answer choices A and B.

Determine the y-intercept: The graph shows that the y-intercept is -2. This eliminates answer choice D.

6. 6

Determine the positive x-intercept: Set equation to 0 and solve.

$$-16(t^2 - 5t - 6) = 0 \ \rightarrow\ t^2 - 5t - 6 = 0 \ \rightarrow$$
$$(t-6)(t+1) = 0 \ \rightarrow\ t = 6 \text{ and } t = -1$$

Hence, $t = 6$ seconds.

7. <u>C</u>

Form a vertex form equation: The time taken to reach the maximum height and the maximum height are the (h, k) coordinates of the parabola vertex.

Time to reach maximum height $= h = 12$.

Maximum height $= k = 2{,}316$.

Hence, $y = a(x-h)^2 + k \rightarrow y = a(x-12)^2 + 2{,}316$.

Determine a: Platform $= 12 = y$-coordinate of the y-intercept. Hence $x = 0$ at this point. Plug this point $(0, 12)$ into the above equation.

$$12 = a(0-12)^2 + 2{,}316 \rightarrow 12 = (144)a + 2{,}316 \rightarrow$$
$$144a = 12 - 2{,}316 = -2{,}304 \rightarrow a = -16$$

Determine height in 16 seconds: Plug $a = -16$, and time $= x = 16$ into the equation.

$$y = -16(16-12)^2 + 2{,}316 = -16(4)^2 + 2{,}316 =$$
$$-16(16) + 2{,}316 = 2{,}060$$

8. <u>D</u>

a is the x-coordinate of the vertex. It represents the unit shoe price for which the earnings were maximum on the opening day.

9. <u>D</u>

Plug the given values into vertex form: Since $k = 22$ is positive and the parabola intersects the x-axis at two points, it must open downwards. Hence, a is negative.

$$y = -a(x-8)^2 + 22$$

Convert to standard form:

$$-a(x^2 - 16x + 64) + 22 = -ax^2 + 16ax - 64a + 22$$

Hence, $a = -a$, $b = 16a$, and $c = -64a + 22$.

$$a + b + c = -a + 16a - 64a + 22 = -49a + 22$$

Evaluate each answer choice: Equate each answer choice to $-49a + 22$. The value of a must be negative.

On such a question, the answer choice will most likely be either A or D.

Answer choice A:

$$-49a + 22 = 10 \rightarrow -49a = -12 \rightarrow -a = \frac{-12}{49}$$

a is positive. This eliminates answer choice A.

Answer choice D:

$$-49a + 22 = 28 \rightarrow -49a = 6 \rightarrow -a = \frac{6}{49} \rightarrow a = -\frac{6}{49}$$

a is negative. This is correct answer choice.

Category 33 – Vertex Form Equation of a Parabola

1. <u>B</u>

Determine if the parabola opens upward or downward: Since a is negative, the parabola opens downward. This eliminates answer choices C and D.

Determine the vertex: The coordinates of the vertex (h, k) are $(s, -t)$. This eliminates answer choice A.

2. <u>D</u>

Determine the coordinates of the vertex from the graph:

$$(2, -10)$$

3. <u>A</u>

Determine the coordinates of the vertex from the graph: The coordinates of the vertex (h, k) in the graph are $(40, 60)$. Hence, $(x-h) = (x-40)$ and $k = 60$. This eliminates answer choices B, C, and D.

4. <u>27</u>

Determine the y-intercept of the y-intercept:

Plug $x = 0$ in the given function and determine y.

$$y = 2(0-8)^2 - 101 = 2(-8)^2 - 101 =$$
$$2(64) - 101 = 128 - 101 = 27$$

5. <u>C</u>

Determine the x-coordinate of the vertex from the graph:

$$x\text{-coordinate of the vertex} = h = -1$$

Hence, $(x-h)^2 = (x-(-1))^2 = (x+1)^2 \rightarrow b = 1$.

6. <u>A</u>

$h = x$ coordinate of the vertex.

Since $x = -5$ and $x = 11$ have the same value of y, the x-coordinate of the vertex is the midpoint of these two values of x.

The midpoint of -5 and 11 is 3. Use the midpoint formula if unsure.

7. <u>6</u>

$x =$ percent discount offered.

$y =$ quarterly profit $= q$.

To determine the quarterly profit when no discount is offered, plug $x = 0$ into the equation and solve for y.

$$y = -(0-4)^2 + 22 \rightarrow$$
$$y = -(16) + 22 = -16 + 22 = 6$$

8. <u>25</u>

Determine the x-coordinate of the vertex:

From the given equation, the x-coordinate of the vertex is

$$(x - 12.5) \rightarrow x = 12.5$$

Determine the x-intercepts:

The x-coordinate of the vertex $= 12.5$ is the midpoint of the two x-intercepts 0 and b. Hence, b must be 25.

Alternatively, use the midpoint formula as shown below.

$x_1 = 0$. $x_2 = b$. Midpoint $= 12.5$.

$$\frac{x_2 + x_1}{2} = \frac{0 + b}{2} = 12.5 \rightarrow b = 12.5 \times 2 = 25$$

9. C

Determine the coordinates of the vertex from the graph:

$$(2, 1)$$

Hence, $(x - h) = (x - 2)$. This eliminates answer choice D. All remaining answer choices have the same vertex. The only difference is the value of a.

Since the parabola opens upward, the equation of the parabola can be written as

$$y = a(x - 2)^2 + 1$$

Determine the value of a: Plug the given point $(3, 4)$ into the above equation for (x, y).

$$4 = a(3 - 2)^2 + 1$$

$$4 = a(1)^2 + 1 \rightarrow 4 = a + 1 \rightarrow a = 4 - 1 = 3$$

This eliminates answer choices A and B.

Category 34 – Factored Form Equation of a Parabola

1. B

Determine the x-intercept:

Plug the point $(-2, -16)$ into the equation.

$$-16 = (-2 + n)(-2 + 6) \rightarrow -16 = (-2 + n)(4) \rightarrow$$

$$-16 = -8 + 4n \rightarrow 4n = -8 \rightarrow n = -2$$

2. D

The two x-intercepts are -15 and 45. The distance is

$$45 - (-15) = 45 + 15 = 60$$

3. A

Determine the x-coordinate of the vertex using the midpoint formula: The two x-intercepts are 5 and 7.

$x_1 = 5$. $x_2 = 7$.

$$\frac{x_2 + x_1}{2} = \frac{7 + 5}{2} = \frac{12}{2} = 6$$

Determine the y-coordinate of the vertex: This is the maximum annual profit. Plug $x = 6$ into the equation.

$$y = -3(6 - 5)(6 - 7) = -3(1)(-1) = 3$$

4. C

Determine the x-coordinate of the vertex using the midpoint formula: The two x-intercepts are 1 and -1.

$x_1 = -1$. $x_2 = 1$.

$$\frac{x_2 + x_1}{2} = \frac{1 - 1}{2} = \frac{0}{2} = 0$$

This eliminates answer choices A and D.

Determine the y-coordinate of the vertex: Plug $x = 0$ into the equation.

$$y = -4(0 - 1)(0 + 1) = -4(-1)(1) = 4$$

This eliminates answer choice B.

5. C

The two x-intercepts from the graph are -2 and 3. Hence, the two factors are $(x + 2)$ and $(x - 3)$.

$$f(x) = k(x + 2)(x - 3)$$

Determine the value of k: Read a point from the graph of the parabola and plug it into the equation. Using point $(0, -3)$ below.

$$-3 = k(0 + 2)(0 - 3) \rightarrow$$

$$-3 = k(2)(-3) \rightarrow -3 = -6k \rightarrow k = \frac{3}{6} = \frac{1}{2}$$

Category 35 – Equivalent Equations of a Parabola

1. C

Determine the x-coordinate of the vertex:

$a = 1$. $b = 6$.

$$-\frac{b}{2a} = -\frac{6}{2 \times 1} = -3$$

Since $x = -3$, the factor is $(x + 3)$. This eliminates answer choices A and B.

Determine the y-coordinate of the vertex: Plug $x = -3$ into the equation.

$$(-3)^2 + (6 \times -3) + 13 = 9 - 18 + 13 = 4$$

This eliminates answer choice D.

2. B

Determine the x-coordinate of the vertex:

Since $a = 0.5$, answer choices C and D can be eliminated.

$a = 0.5$. $b = 4$.

$$-\frac{b}{2a} = -\frac{4}{2 \times 0.5} = -\frac{4}{1} = -4$$

Since $x = -4$, the factor is $(x + 4)$. This eliminates answer choice A.

3. D

Determine the x-coordinate of the vertex:

Since $a = 1.25$, answer choice A can be eliminated.

$a = 1.25$. $b = 7.5$.

$$-\frac{7.5}{2 \times 1.25} = -\frac{7.5}{2.5} = -3$$

Since $x = -3$, the factor is $(x + 3)$. This eliminates answer choice B.

Determine the y-coordinate of the vertex: Plug $x = -3$ into the equation.

$$1.25(-3)^2 + (7.5 \times -3) + 1 =$$

$$11.25 - 22.5 + 1 = -10.25$$

This eliminates answer choice C.

4. C

Determine the x-coordinate of the vertex: Since a is negative, answer choice D can be eliminated.

The x-coordinate of the vertex is the midpoint of -2 and 7 is 2.5. Hence, the factor is $(x-2.5)$. This eliminates answer choice A. Use the midpoint formula, if unsure.

Determine the y-coordinate of the vertex: Plug $x = 2.5$ into the given equation.

$$y = -(2.5-7)(2.5+2) = -(-4.5)(4.5) = 20.25$$

This eliminates answer choice B.

5. A

Since a is negative, answer choice D can be eliminated.

FOIL:

$$-2(x^2+4x+4)+1 = -2x^2-8x-8+1 =$$
$$-2x^2-8x-7$$

Since $c = -7$, answer choices B and C can be eliminated.

6. 12

FOIL $(x+2r)(x-2s)$:

$$x^2-2sx+2rs-4rs \rightarrow$$
$$x^2-2(s-r)x-4rs$$

Determine $s-r$: Equate the coefficients of x.

$$-2(s-r) = -24 \rightarrow s-r = 12$$

Category 36 – Parabola Intersections and Systems of Equations

1. A

Equate the two equations:

$$2x^2-2 = 96$$

Create one quadratic equation.

$$2x^2-2-96 = 0 \rightarrow 2x^2-98 = 0 \rightarrow x^2-49 = 0$$

Determine the values of x:

$$x^2 = 49 \rightarrow x^2 = 7^2 \rightarrow x = \pm 7$$

Answer choice A is one of the values.

2. 235

Determine the value of y:

Since $x = 5$, it can be directly substituted into $y = (10+x)^2+x$.

$$y = (10+5)^2+5 = 15^2+5 = 225+5 = 230$$

Determine $x+y$: $5+230 = 235$.

3. C

Equate the two equations:

Convert the equations to the correct form.

$$y+4x = 2x^2+1 \rightarrow y = 2x^2-4x+1 \rightarrow$$
$$y = (x-1)(x-2) \rightarrow y = x^2-3x+2$$

Equate and create one quadratic equation.

$$2x^2-4x+1 = x^2-3x+2$$
$$2x^2-4x+1-x^2+3x-2 = 0 \rightarrow x^2-x-1 = 0$$

Determine number of solutions using the discriminant:

$a = 1.\ b = -1.\ c = -1.$

$$b^2-4ac = (-1)^2-(4\times 1\times -1) = 1+4 = 5$$

Since the discriminant is greater than 0, there are two solutions.

4. 9

$y = 9$ is a horizontal line. Since it intersects the parabola at one point, it must be tangent to the vertex of the parabola. Hence, the y-coordinate n is also 9.

5. 1

Equate the two equations:

Convert the equations to the correct form.

$$2y = 4x+8 \rightarrow y = 2x+4$$

Equate and create one quadratic equation.

$$2x+4 = -ax^2-4x-5 \rightarrow$$
$$2x+4+ax^2+4x+5 = 0 \rightarrow ax^2+6x+9 = 0$$

Determine the value of a using the discriminant: Since the parabola and the line intersect at one point, the discriminant is 0.

$a = a.\ b = 6.\ c = 9.$

$$b^2-4ac = 0 \rightarrow (6)^2-(4\times a\times 9) = 0 \rightarrow$$
$$36-36a = 0 \rightarrow 36 = 36a \rightarrow a = 1$$

6. 3

Convert the equations to the correct form.

$$y-kx = 1 \rightarrow y = kx+1$$

The slope of a line in the form $y = mx+b$, is the m value (refer to section 1 for further details, if needed).

Hence, the slope of line l is k.

Equate and create one quadratic equation.

$$x^2-kx+10 = kx+1 \rightarrow$$
$$x^2-kx+10-kx-1 = 0 \rightarrow x^2-2kx+9 = 0$$

Set the discriminant to 0 and determine k: Since the two graphs intersect at one point, the discriminant is 0.

$a = 1.\ b = -2k.\ c = 9.$

$$b^2-4ac = 0 \rightarrow (-2k)^2-(4\times 1\times 9) = 0 \rightarrow$$
$$4k^2-36 = 0 \rightarrow 4k^2 = 36 \rightarrow k^2 = 9 \rightarrow k = \pm 3$$

Since the slope of line l is positive, $k = 3$.

7. 8.8

Equate the two equations:

$$3x = 2x^2 + 11.8x$$

Create one quadratic equation.

$$2x^2 + 11.8x - 3x = 0 \rightarrow 2x^2 + 8.8x = 0$$

Factor to determine the value of $x = a$:

$2x(x + 4.4) = 0 \rightarrow x + 4.4 = 0$ and $2x = 0$

$$a = x = -4.4$$

Determine the value of $y = b$: Plug $x = -4.4$ into either of the given equations.

$$b = y = 3(-4.4) = -13.2$$

Determine $a - b$:

$$(-4.4) - (-13.2) = -4.4 + 13.2 = 8.8$$

8. B

Equate the two equations:

Convert the equations to the correct form.

$$x - y = -5 \rightarrow y = x + 5$$

$$-x + y = -2x^2 + 9 \rightarrow y = -2x^2 + x + 9$$

Equate and create one quadratic equation.

$$-2x^2 + x + 9 = x + 5 \rightarrow$$

$$-2x^2 + x + 9 - x - 5 = 0 \rightarrow -2x^2 + 4 = 0$$

Determine the value of x: Factor.

$$2x^2 - 4 = 0 \rightarrow 2x^2 = 4 \rightarrow x^2 = 2 \rightarrow x = \pm\sqrt{2}$$

In answer choice B, $x = \sqrt{2}$. None of the other answer choice have $x = -\sqrt{2}$ or $x = \sqrt{2}$. This eliminates answer choices A, C, and D.

Category 37 – Graph Transformations of a Parabola

1. D

Determine the reflection:

Reflection across the y-axis is $y = 2(-x + 3)^2 - 1$

2. B

Determine the function $g(x)$:

$g(x) = f(x) - 5$ is a downward translation of $g(x)$ by 5 units. Hence, $g(x) = 2x^2 - 7 - 5 = 2x^2 - 12$.

Determine the table containing points on $g(x)$:

Start with the x-value of the point from the table in the first answer choice. Plug it in the definition of $g(x)$ and evaluate the corresponding y-value. It must match with the $g(x)$ value in the table. The table that contains all three correct x and y-values is the correct answer choice.

Only the points from answer choices B are correct.

3. C

Determine the translated equation:

Translation of 11 units left is

$$y = 2(x - 3 + 11)^2 - 8 \rightarrow y = 2(x + 8)^2 - 8$$

Reflection of above graph across x-axis is

$$y = 2(-x + 8)^2 - 8$$

4. A

Determine the translation:

$x^2 + 4$ to $(x - 4)^2$ is a translation of 4 units right and 4 units down.

This eliminates answer choices B, C, and D.

5. 49

Determine the function $g(x)$:

$$g(x) = f(x + 7) = 2(x + 7)^2 - 8(x + 7) + 7$$

Determine $g(0)$: Plug in $x = 0$ and determine y.

$$g(0) = 2(0 + 7)^2 - 8(0 + 7) + 7 =$$

$$2(49) - 8(7) + 7 = 98 - 56 + 7 = 49$$

6. 12

Determine the function $f(x)$:

7 units right of $g(x)$ is $g(x - 7)$. Hence,

$$f(x) = g(x - 7) = -\frac{1}{2}(x - 1 - 7)(x - 9 - 7) =$$

$$= -\frac{1}{2}(x - 8)(x - 16)$$

Determine x-coordinate of the vertex of $g(x)$:

The two factors of $g(x)$ are 8 and 16. The midpoint is 12. Alternatively, use the midpont formula.

Category 38 – Equivalent Quadratic Expressions

1. C

Equate the coefficients and constants of both sides: It is given that both the expressions are equal. Hence,

$$7(x^2 + 0.04c) - x^2 = 0.2ax^2 + 0.28c \rightarrow$$

$$7x^2 + 0.28c - x^2 = 0.2ax^2 + 0.28c \rightarrow$$

$$6x^2 + 0.28c = 0.2ax^2 + 0.28c$$

Equate coefficients of x^2.

$$6 = 0.2a \rightarrow a = \frac{6}{0.2} = 30$$

2. A

Equate the coefficients and constants of both sides: The expressions on both sides of the equation must be equal.

The right-side expression does not have the variable x. Hence, $4cx = 0 \rightarrow c = 0$

3. D

Equate the coefficients and constants of both sides: It is given that both the expressions are equal. Hence,

$$(x+3)(ax+c) = 6x^2 + bx + 9$$

FOIL the left-side expression.

$$ax^2 + cx + 3ax + 3c = 6x^2 + bx + 9 \;\rightarrow$$
$$ax^2 + (c+3a)x + 3c = 6x^2 + bx + 9$$

Equate.

$a = 6$, $b = c + 3a$.

$$3c = 9 \;\rightarrow\; c = 3$$

Determine b by substituting $a = 6$ and $c = 3$.

$$b = c + 3a = 3 + (3 \times 6) = 3 + 18 = 21$$

4. A

Equate the coefficients and constants of both sides:

$$(1+k)x^2 + c = 4cx^2 + ax - 0.25$$

Hence, $c = -0.25$.

Determine k: Equate coefficients of x^2 and substitute $c = -0.25$.

$$1 + k = 4c \;\rightarrow\; 1 + k = 4(-0.25) \;\rightarrow$$
$$1 + k = -1 \;\rightarrow\; k = -1 - 1 = -2$$

Determine a: Since the left-side does not have variable x, $a = 0$.

Determine $a + k$:

$$-2 + 0 = -2$$

5. 5

Equate the coefficients and constants of both sides: The expressions on both sides of the equation must be equal.

FOIL the left-side expression.

$$4x^2 + 2x + 6x + 3 = 4x^2 + 8x + c - 2 \;\rightarrow$$
$$4x^2 + 8x + 3 = 4x^2 + 8x + c - 2$$

Equate the constants: $c - 2 = 3 \;\rightarrow\; c = 5$

6. B

Evaluate the coefficients and constants of both sides: The expressions on both sides of the equation must be equal.

FOIL the left-side expression.

$$3abx^2 + 3acx + 4bx + 4c = 6x^2 + 4x$$

Since there is no constant in the right-side expression, $c = 0$. This eliminates answer choices C and D.

Evaluate answer choice A: $a = 1$ and $b = 2$.

$$(3 \times 1 \times 2)x^2 + 0 + (4 \times 2)x + 0 = 6x^2 + 4x \;\rightarrow$$
$$6x^2 + 8x = 6x^2 + 4x$$

The expressions on both sides are not equal. This eliminates answer choice A.

In the correct answer choice B, both sides are equal:

$$(3 \times 2 \times 1)x^2 + 0 + (4 \times 1)x + 0 = 6x^2 + 4x \;\rightarrow$$
$$6x^2 + 4x = 6x^2 + 4x$$

7. D

Since the right-side expression does not have any variable, the equation will have no solution.

Section 6 – Drill

1. C

Determine the y-coordinate of the y-intercept of the parabola in the graph: $c = 7$

2. C

Minimum value is y-coordinate of a parabola opening upward. Since the parabola opens upward, a is positive. This eliminates answer choices A and D. Of the remaining answer choices, only choice C has $k = -12$.

3. A

Determine the roots:

$$x^2 + 4x - 96 = 0 \;\rightarrow\; (x-8)(x+12) = 0$$

Hence, the two x-intercepts are $(0, 8)$ and $(0, -12)$.

4. D

The maximum weekly profit is the y-coordinate of the vertex. The graph shows that the y-coordinate $= 400$.

5. B

Since $h(-5) = h(19)$, the y-coordinates of $x = -5$ and $x = 19$ are the same. Hence, the midpoint of $x = -5$ and $x = 19$ is the x-coordinate of the vertex $= 7$. Use the midpoint formula, if unsure.

6. B

Determine the x-coordinate of the vertex:

$a = 1$. $b = -8$.

$$-\frac{b}{2a} = -\frac{-8}{2 \times 1} = 4$$

Since $x = 4$, the factor is $(x - 4)$. This eliminates answer choices B and D.

Determine the y-coordinate of the vertex: Plug $x = 4$ into the equation.

$$(4)^2 - (8 \times 4) + 13 = 16 - 32 + 13 = -3$$

This eliminates answer choice C.

7. C

Evaluate each answer choice for the midpoint of the two x-intercepts: The axis of symmetry is the midpoint of the two x-intercepts.

Answer choices A and D can be eliminated since both these answer choices have two identical x-intercepts.

Answer choice B: The two x-intercepts are 1 and -5. Their midpoint is -2. This eliminates answer choice B.

In the correct answer choice C, the two x-intercepts are -1 and 5 and their midpoint is 2.

8. 4

Determine the x-intercepts of the parabola:

Set $y = 0$ in the equation and solve.

$$-16t^2 + 48t + 64 = 0 \rightarrow -16(t^2 - 3t - 4) = 0$$

$$t^2 - 3t - 4 = 0 \rightarrow (t-4)(t+1) = 0$$

The two x-intercepts are

$$(t-4) \rightarrow t = 4 \text{ and } (t+1) \rightarrow t = -1$$

The correct answer is the positive x-intercept = 4.

9. C

Determine the x-coordinate of the vertex:

Since $a = -2$, answer choice D can be eliminated.

The two x-intercepts are -1 and 9. Their midpoint is 4. (Use midpoint formula, if unsure.)

Hence, the factor is $(x - 4)$.

This eliminates answer choice A.

Determine the y-coordinate of the vertex: Plug in $x = 4$.

$$-2(4+1)(4-9) = -2(5)(-5) = 50$$

This eliminates answer choice B.

10. 2

Use the discriminant:

$a = a.\ \ b = -8.\ \ c = 8.$

Since the equation has one solution, discriminant = 0.

$$(-8)^2 - (4 \times a \times 8) = 0 \rightarrow 64 - 32a = 0 \rightarrow a = 2$$

11. D

Compare the vertex of the two functions:

Vertex of $f(x)$ is at (1, 2).

Vertex of $g(x)$ is at (−2, −1).

Hence, the vertex of $g(x)$ is 3 units left and 3 units below the vertex of $f(x)$.

12. 12

Determine the factors: First divide the equation by 2.

$$2x^2 + kx + 10 = 0 \rightarrow x^2 + \frac{k}{2}x + 5 = 0$$

The two multiples of $c = 5$ are 1 and 5, and −1 and −5. Since it is given that $(x + 5)$ is a factor of the equation, the other factor must be $(x + 1)$. Hence, the given equation can be factored as $(x + 5)(x + 1)$.

$$x^2 + \frac{k}{2}x + 5 = (x+5)(x+1)$$

FOIL the right-side of the equation.

$$x^2 + \frac{k}{2}x + 5 = x^2 + 6x + 5$$

Equate the coefficients of x.

$$\frac{k}{2} = 6 \rightarrow k = 12$$

13. 30

$g(x)$ can be rewritten as

$$g(x) = -(k+x)(x-12) = -(x+k)(x-12)$$

Determine k: Since the midpoint point of the two x-intercepts is 3.5, both are equidistant from 3.5. From the given equation, one of the x-intercepts is 12.

The distance between 3.5 and 12 is $12 - 3.5 = 8.5$.

Since, 12 is higher than 3.5, k must be 8.5 less than 3.5. Hence, $k = 3.5 - 8.5 = -5$. The factor is $(x + 5)$.

Determine $g(x)$: Substitute $(x + 5)$ for $(x + k)$.

$$g(x) = -(x+5)(x-12)$$

Determine $g(10)$: $-(10+5)(10-12) = 30$

14. A

Form a quadratic equation:

$$4x^2 + 4x - 2 = 3x^2 + 9x - 5 \rightarrow$$

$$4x^2 + 4x - 2 - 3x^2 - 9x + 5 = 0 \rightarrow x^2 - 5x + 3 = 0$$

Determine the roots: $a = 1.\ \ b = -5.\ \ c = 3.$

$$\frac{-(-5) \pm \sqrt{(-5)^2 - (4 \times 1 \times 3)}}{2 \times 1} = \frac{5 \pm \sqrt{25 - 12}}{2} =$$

$$\frac{5 \pm \sqrt{13}}{2} = 2.5 \pm \frac{\sqrt{13}}{2}$$

Answer choice A is one of the solutions.

15. C

Equate the two equations:

Convert the equations to the correct form.

$$x - y = -4 \rightarrow y = x + 4$$

Equate and create one quadratic equation.

$$x + 4 = x^2 - 5x + 4$$

$$x^2 - 5x + 4 - x - 4 = 0 \rightarrow x^2 - 6x = 0$$

Determine the values of x: Factor out x.

$$x(x-6) = 0$$

The two values of x are $x = 0$ and $x = 6$.

This eliminates answer choices A and D.

Determine the value of y: Plug $x = 6$ into any of the equations.

$$x + 4 \rightarrow 6 + 4 = 10$$

This eliminates answer choice B.

16. A

Determine the translated function $f(x + 16)$:

$$y = f(x+16) = (x - 1 + 16)(x - 7 + 16) =$$
$$(x+15)(x+9)$$

The two x-intercepts are −15 and −9.

Determine the x-coordinate of the vertex: x-coordinate of the vertex is the midpoint of the two x-intercepts = −12.

Determine the y-coordinate of the vertex: Plug $x = -12$ into $f(x + 16)$.

$$y = (-12 + 15)(-12 + 9) = (3)(-3) = -9$$

17. A

Convert to the correct form:

$$9x^2 + 6 = 7x^2 + 3x \rightarrow 2x^2 - 3x + 6 = 0$$

Determine number of solutions using the discriminant:

$a = 2.\ b = -3.\ c = 6.$

$$b^2 - 4ac = (-3)^2 - (4 \times 2 \times 6) = 9 - 48 = -39$$

Since discriminant < 0, there is no real solution.

18. 2

Determine m: Plug $(-0.8, m)$ into the equation.

$$m = 5(-0.8)^2 - 0.8 = 5(0.64) - 0.8 = 2.4$$

Hence, $(-0.8, m) = (-0.8, 2.4)$.

Determine n: Plug $(1, n)$ into the equation.

$$n = 5(1)^2 + 1 = 5(1) + 1 = 6$$

Hence, $(1, n) = (1, 6)$.

Determine the slope of line k: use the above two points.

$$\frac{6 - 2.4}{1 - (-0.8)} = \frac{3.6}{1.8} = 2$$

19. D

Use the sum formula: Rearrange before using the formula.

$$15x^2 - 2ax + 10bx - ab = 0 \rightarrow$$

$$15x^2 - 2(a - 5b)x - ab = 0$$

$a = 15.\ b = -2(a - 5b).\ c = -ab.$

$$-\frac{b}{a} = -\frac{-2(a - 5b)}{15} = \frac{2(a - 5b)}{15}$$

Equate with $(a - 5b)c$ and determine c:

$$(a - 5b)c = \frac{2(a - 5b)}{15} \rightarrow c = \frac{2}{15}$$

20. D

Determine b in terms of a:

$(5, 0)$ and $(-17, 0)$ are the x-intercepts. Their midpoint $= x$-coordinate of the vertex $= -6$. Hence,

$$-\frac{b}{2a} = -6 \rightarrow -b = -6 \times 2a \rightarrow b = 12a$$

Evaluate $a - b$: Substitute $b = 12a$ in $a - b$.

$$a - b = a - 12a = -11a$$

If $a = -2$, then $a - b = -11 \times -2 = 22$. Hence, for $a < -2$, $a - b$ must be greater than 22.

This eliminates answer choices A, B, and C.

21. D

Equate the coefficients of both sides: The expressions on both sides of the equation must be equal.

$$abx^2 + 4ax + 3bx + 12 = 10x^2 + kx + 12 \rightarrow$$

$$abx^2 + (4a + 3b)x + 12 = 10x^2 + kx + 12$$

Equate: $ab = 10$ and $4a + 3b = k$.

Get the values of a and b: a and b are 2 multiples of 10, and greater than 0. Hence, ignore the negative multiples.

The multiples of 10 are 1 and 10 or 2 and 5. Since it is given that $a + b = 7$, the two multiples must be 2 and 5.

Hence, the possible values of a and b are

$$a = 2 \text{ and } b = 5 \text{ or } a = 5 \text{ and } b = 2.$$

Plug the above possible combinations into the equation $4a + 3b = k$.

For $a = 2$ and $b = 5$: $k = (4 \times 2) + (3 \times 5) = 23$.

For $a = 5$ and $b = 2$: $k = (4 \times 5) + (3 \times 2) = 26$.

22. 37

Use the sum formula:

$a = \frac{1}{54}.\ b = -\left(2 - \frac{k}{18}\right).$

$$-\frac{b}{a} = -3 \rightarrow -\left(-\left(2 - \frac{k}{18}\right) \div \frac{1}{54}\right) = -3 \rightarrow$$

$$\left(\frac{36 - k}{18}\right) \times \frac{54}{1} = -3 \rightarrow (36 - k) \times 3 = -3 \rightarrow k = 37$$

23. B

Evaluate a: Since $f(-15) = f(7)$, the x-coordinate of the vertex is the midpoint of -15 and 7.

Midpoint $= -4$ ($= h$ in the vertex form).

From the equation, $b = 9$. Use the formula below, and equate to -4 to determine a.

$$-\frac{b}{2a} = -4 \rightarrow -\frac{9}{2a} = -4 \rightarrow -9 = -8a \rightarrow a = \frac{9}{8}$$

Option I and answer choices A and C can be eliminated since a cannot be less than or equal to 1.

Evaluate c: The only other given information is that $k > 0$. Create a vertex form equation, plug the above values of a and h, and convert it to the standard form.

$$f(x) = \frac{9}{8}(x + 4)^2 + k = \frac{9}{8}(x^2 + 8x + 16) + k =$$

$$\frac{9}{8}x^2 + 9x + 18 + k$$

Hence, $c = 18 + k \rightarrow k = c - 18$. Since k is greater than 0, c must be greater than 18. Option II is correct.

24. B

Equate the two equations:

Convert the equations to the correct form.

$$x + y = -a \rightarrow y = -x - a$$

$$x = y - 2x^2 + 2 \rightarrow y = 2x^2 + x - 2$$

Equate and create one quadratic equation.

$$2x^2 + x - 2 = -x - a \rightarrow$$

$$2x^2 + x - 2 + x + a = 0 \rightarrow 2x^2 + 2x - 2 + a = 0$$

Set the discriminant to 0 and determine a:

$$b^2 - 4ac = 0 \rightarrow (2)^2 - \left(4 \times 2 \times (-2 + a)\right) = 0 \rightarrow$$

$$4 - \left(8 \times (-2 + a)\right) = 0 \rightarrow 4 - (-16 + 8a) = 0 \rightarrow$$

$$4 + 16 - 8a = 0 \rightarrow 20 = 8a \rightarrow a = 2.5$$

Plug $a = 2.5$ and determine x:

$$2x^2 + 2x - 2 + 2.5 = 0 \rightarrow 2x^2 + 2x + 0.5 = 0 \rightarrow$$

$$x^2 + x + 0.25 = 0 \rightarrow (x + 0.5)^2 = 0 \rightarrow$$

$$x = -0.5 = -\frac{1}{2}$$

Section 7 – Absolute Value

Category 39 – Absolute Value and Equations

1. A

Move 8 to the right side of the equation.

$$|x-4| = 6-8 \;\rightarrow\; |x-4| = -2$$

Absolute values cannot be negative. Hence, there is no solution.

2. C

Move 2 to the right side of the equation.

$$|2x-7| = 13+2 \;\rightarrow\; |2x-7| = 15$$

Solve for the positive value:

$$2x-7 = 15 \rightarrow 2x = 15+7 \;\rightarrow\; 2x = 22 \;\rightarrow x = 11$$

Solve for the negative value:

$$2x-7 = -15 \;\rightarrow\; 2x = -15+7 \;\rightarrow$$
$$2x = -8 \;\rightarrow\; x = -4$$

Check for an extraneous solution: -4 is not an extraneous solution.

Solve for $|b|$: Since $a > b$, $a = 11$ and $b = -4$.

$$|-4| = 4$$

3. 8

Move 3 to the right side of the equation.

$$|p-4| = 6-3 \;\rightarrow\; |p-4| = 3$$

Solve for the positive value:

$$p-4 = 3 \;\rightarrow\; p = 3+4 \;\rightarrow\; p = 7$$

Solve for the negative value:

$$p-4 = -3 \;\rightarrow\; p = -3+4 \;\rightarrow\; p = 1$$

Check for an extraneous solution: 1 and 7 are not extraneous solutions.

Add the two values:

$$7+1 = 8$$

4. 5

Solve for the positive value:

$$2x+5 = 7 \;\rightarrow\; 2x = 7-5 \;\rightarrow\; 2x = 2 \;\rightarrow\; x = 1$$

Solve for the negative value:

$$2x+5 = -7 \;\rightarrow\; 2x = -7-5 \;\rightarrow$$
$$2x = -12 \;\rightarrow\; x = -6$$

Check for an extraneous solution: -6 and 1 are not extraneous solutions.

Solve for $|m+n|$: Either value of x could be m or n. Same results will be obtained with either combination.

$$|1-6| = |-5| = 5 \text{ or } |-6+1| = |-5| = 5$$

5. A

Solve for the positive value:

$$3b-6 = 2b-b \;\rightarrow\; 3b-6 = b \;\rightarrow$$
$$3b-b = 6 \;\rightarrow\; 2b = 6 \;\rightarrow\; b = 3$$

This is not an answer choice.

Solve for the negative value:

$$3b-6 = -(2b-b\,) \;\rightarrow\; 3b-6 = -b \;\rightarrow$$
$$3b+b = 6 \;\rightarrow\; 4b = 6 \rightarrow\; b = 1.5$$

Check for an extraneous solution: 1.5 is not an extraneous solution. It is one of the answer choices.

6. 2, 4, 8, or 16

Equation $|x+3| = 1$

Solve for the positive value:

$$x+3 = 1 \;\rightarrow\; x = 1-3 \;\rightarrow\; x = -2$$

Equation $|2y-3| = 5$

Solve for the positive value:

$$2y-3 = 5 \;\rightarrow\; 2y = 5+3 \;\rightarrow\; 2y = 8 \;\rightarrow\; y = 4$$

Check for an extraneous solution: None are extraneous solutions.

Solve for $|xy|$:

$$|-2\times 4| = |-8| = 8$$

(For the purpose of completion, see below for other solutions.

The other value of x is

$$x+3 = -1 \;\rightarrow\; x = -1-3 \;\rightarrow\; x = -4$$

The other value of y is

$$2y-3 = -5 \;\rightarrow\; 2y = -5+3 \;\rightarrow\; y = -1$$

Hence, all other possible $|xy|$ values are

$$|-2\times -1| = |2| = 2$$
$$|-4\times 4| = |-16| = 16$$
$$|-4\times -1| = |4| = 4$$

)

7. 1

Move -5 to the right-side of the equation.

$$|9-x| = \frac{-40x}{-5} \;\rightarrow\; |9-x| = 8x$$

Solve for the positive value:

$$9-x = 8x \;\rightarrow\; 8x+x = 9 \;\rightarrow\; 9x = 9 \;\rightarrow\; x = 1$$

(The solution to the negative value, $9-x = -8x$, is an extraneous solution. Desmos graphing calculator will only graph $x = 1$.)

8. <u>44</u>

Determine $f(-2)$: Plug $x = -2$.

$$f(-2) = |-2-5|^2 - |-2-7| = |-7|^2 - |-9| = 49 - 9 = 40$$

Determine $f(2)$: Plug $x = 2$.

$$f(2) = |2-5|^2 - |2-7| = |-3|^2 - |-5| = 9 - 5 = 4$$

Determine $f(-2) + f(2)$:

$$40 + 4 = 44$$

9. <u>1 or 7</u>

<u>Equation $|x + 5| = 3$</u>

Solve for the positive value:

$$x + 5 = 3 \ \rightarrow \ x = 3 - 5 \ \rightarrow \ x = -2$$

<u>Equation $|y + 2| = 3$</u>

Solve for the positive value:

$$y + 2 = 3 \ \rightarrow \ y = 3 - 2 \ \rightarrow \ y = 1$$

Since question asks for $y > 0$, $y = 1$ meets the condition.

Check for an extraneous solution: None are extraneous solutions.

Solve for $|x + y|$:

$$|-2 + 1| = |-1| = 1$$

For the purpose of completion, see below for other possible solutions.

The second value of x is

$$x + 5 = -3 \ \rightarrow \ x = -3 - 5 \ \rightarrow \ x = -8$$

The second value of y is

$$y + 2 = -3 \ \rightarrow \ y = -3 - 2 \ \rightarrow \ y = -5$$

This does not meet $y > 0$ condition and can be eliminated.

Hence, the other possible $|x + y|$ value is

$$|-8 + 1| = |-7| = 7$$

Category 40 – Absolute Value and Inequalities

1. <u>C</u>

Determine the solution set:

$$-2 < x + 3 < 2 \ \rightarrow \ -2 - 3 < x + 3 - 3 < 2 - 3 \ \rightarrow \ -5 < x < -1$$

The 3 possible values of x are -2, -3, -4.

2. <u>A</u>

Determine the midpoint of the two numbers:

$$\frac{100 + 150}{2} = \frac{250}{2} = 125$$

This eliminates answer choices C and D.

Determine the distance of the two numbers from the midpoint:

Both the numbers are at a distance of 25 from the midpoint. This eliminates answer choice B.

3. <u>C</u>

Determine the solution set:

Since the absolute value has greater than inequality symbol, the values of a can be

$$a > 4 \text{ or } a < -4$$

Hence, answer options I and II are correct.

4. <u>D</u>

Determine the solution set:

$$-34{,}000 \le r - 79{,}000 \le 34{,}000 \ \rightarrow$$
$$-34{,}000 + 79{,}000 \le r \le 34{,}000 + 79{,}000 \ \rightarrow$$
$$45{,}000 \le r \le 113{,}000$$

The maximum value of $r = 113{,}000$.

When using Desmos graphing calculator, type x for r.

5. <u>B</u>

Determine the solution set:

$$-4 < x - 1 < 4 \ \rightarrow \ -4 + 1 < x - 1 + 1 < 4 + 1 \ \rightarrow \ -3 < x < 5$$

6. <u>D</u>

Determine the solution set:

Since the absolute value has greater than inequality symbol, the values of x can be

$$2x + 3 > 5 \text{ or } 2x + 3 < -5$$

Solve $2x + 3 > 5$:

$$2x > 5 - 3 \ \rightarrow \ 2x > 2 \ \rightarrow \ x > 1$$

Solve $2x + 3 < -5$:

$$2x < -5 - 3 \ \rightarrow \ 2x < -8 \ \rightarrow \ x < -4$$

Hence, the possible values of x can be greater than 1 or less than -4. The integers 1, 0, -1, -2, -3, -4 cannot be the values. Options I and III are correct.

Category 41 – Absolute Value and Functions

1. A

Determine the shift in the graph:

The vertex of the graph is below 0 and x-coordinate $= 0$. Hence, the y-coordinate of the vertex must be negative, and the x-coordinate is not shifted horizontally. This eliminates answer choices B, C, and D.

2. C

Determine the shift in the graph:

The vertex of the graph is below 0 and to the left. Hence, the x- and y-coordinates of the vertex must be negative. This eliminates answer choices A, B, and D.

3. B

Determine the shift in the graph:

$x - 1$ indicates that the vertex is shifted horizontally by 1 unit to the right. This eliminates answer choices A and D.

y-coordinate $= 1$ indicates that the vertex is shifted up 1 unit. This eliminates answer choice C.

4. A

Determine the change in y-values:

All the y-values must become positive. Note that not every graph that has positive y-values is the correct graph. The negative y-values must match the corresponding positive y-values (as shown in key points). The correct answer choice is A.

5. B

Evaluate each answer choice: Since $|f(t)|$ is positive, $f(t)$ must also be positive. Hence, for each value of t in the answer choice, check for the value of y on the graph. The correct answer choice will have a positive y-value.

Answer choice A: The y-value for $x = -4$ is not on the graph of the function.

Answer choice B: For $x = -2$, $y = 3$.

Answer choice C: For $x = 3$, $y = -1$.

Answer choice D: For $x = 4$, $y = -3$.

Only answer choice B has a positive value of y.

6. C

Read the value of $f(x)$ from the table:

$$f(1) = -2$$
$$f(3) = -1$$

Solve for $|2f(1) - f(3)|$:

$$|(2 \times -2) - (-1)| = |-4 + 1| = |-3| = 3$$

7. D

Plug $x = 5$ into the function g:

$$g(5) = (3 \times 5) - 10 = 15 - 10 = 5$$

Solve for $|f(-2) + g(5)|$: It is given that $f(-2) = -10$.

$$|-10 + 5| = |-5| = 5$$

Section 7 – Drill

1. 2

Solve for the positive value:

$$6 - x = 4 \rightarrow x = 6 - 4 \rightarrow x = 2$$

Since it is given that $s = 10$, t must be 2.

2. C

Determine the solution set:

Since the absolute value has greater than inequality symbol, the values of x can be

$$x - 3 > 5 \text{ or } x - 3 < -5$$

Solve for $x - 3 > 5$:

$$x > 5 + 3 \rightarrow x > 8$$

Solve for $x - 3 < -5$:

$$x < -5 + 3 \rightarrow x < -2$$

Hence, the possible values of x can be greater than 8 or less than -2. The integers $-2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8$ cannot be the values.

This eliminates answer choices A ($x = |-1| = 1$), B ($x = |-3| = 3$), and D ($x = 6$).

3. 7

Move 1 to the right side of the equation.

$$|2x + 7| = 4 - 1 \rightarrow |2x + 7| = 3$$

Solve for the positive value:

$$2x + 7 = 3 \rightarrow 2x = 3 - 7 \rightarrow 2x = -4 \rightarrow x = -2$$

Solve for the negative value:

$$2x + 7 = -3 \rightarrow 2x = -3 - 7 \rightarrow$$
$$2x = -10 \rightarrow x = -5$$

Check for an extraneous solution: -2 and -5 are not extraneous solutions.

Solve for $|a + b|$: Either value of x could be a or b. Same results will be obtained with either combination.

$$|-2 - 5| = |-7| = 7$$

4. B

Determine the solution set:

$$-16 \leq l - 38 \leq 16 \rightarrow$$
$$-16 + 38 \leq l \leq 16 + 38 \rightarrow 22 \leq l \leq 54$$

The smallest value of $l = 22$.

When using Desmos graphing calculator, type x for l.

5. C

Determine the shift in the graph:

The vertex of the graph is below 0 and to the right. Hence, the x-coordinate of the vertex must be positive, and the y-coordinate of the vertex must be negative. This eliminates answer choices A, B, and D.

6. D

Determine the solution set:

$$-3 < 2x - 1 < 3 \rightarrow$$
$$-3 + 1 < 2x - 1 + 1 < 3 + 1 \rightarrow$$
$$-2 < 2x < 4 \rightarrow -1 < x < 2$$

7. A

Determine the midpoint of the two numbers:

$$\frac{40 + 70}{2} = \frac{110}{2} = 55$$

This eliminates answer choices B and D.

Determine the distance of the numbers from the midpoint:

Both the numbers are at a distance of 15 from the midpoint. This eliminates answer choice C.

8. B

Move -9 to the right side of the equation.

$$|9 - x| = \frac{-45x}{-9} \rightarrow |9 - x| = 5x$$

Solve for the positive value:

$$9 - x = 5x \rightarrow 5x + x = 9 \rightarrow 6x = 9 \rightarrow$$
$$x = \frac{9}{6} = \frac{3}{2} = 1.5$$

Solve for the negative value:

$$9 - x = -5x \rightarrow -5x + x = 9 \rightarrow -4x = 9 \rightarrow$$
$$x = -\frac{9}{4} = -2.25$$

Check for an extraneous solution: Plug $x = -2.25$.

$$-9|9 - (-2.25)| = -45(-2.25) \rightarrow$$
$$-9|11.25| = 101.25 \rightarrow -101.25 = 101.25$$

Hence, $x = -2.25$ is an extraneous solution.

Check for an extraneous solution: Plug $x = 1.5$.

$$-9|9 - (1.5)| = -45(1.5) \rightarrow -9|7.5| = -67.5 \rightarrow$$
$$-67.5 = -67.5$$

Hence, one solution.

9. C

Plug the given values of x into the function:

$$g(3) = 3^2 - (2 \times 3) - 11 = 9 - 6 - 11 = -8$$
$$g(5) = 5^2 - (2 \times 5) - 11 = 25 - 10 - 11 = 4$$

Solve $|g(3) - g(5)|$:

$$|-8 - 4| = |-12| = 12$$

10. B

Determine $f(-5)$: Plug $x = -5$.

$$f(-5) = |2(-5)^3 + 4(-5)| - |4(-5) - 4|^2$$
$$= |2(-125) - 20| - |-20 - 4|^2$$
$$= |-250 - 20| - |-24|^2 = |-270| - |-24|^2 =$$
$$270 - 576 = -306$$

Determine a:

$$f(-5) + g(a) = -276 \rightarrow -306 + |2a| = -276 \rightarrow$$
$$|2a| = -276 + 306 = 30$$

Remove bars and solve for the two values of a:

$$2a = 30 \rightarrow a = 15$$
$$2a = -30 \rightarrow a = -15$$

-15 is answer choice B.

11. D

Read the value of y from the graph for $x = 3$:

$$y = -3$$

Read the values of x from the graph for $y = -3$:

The two values of x that define $f(3)$ are $x = -4$ and $x = 3$ (given).

Sum of the absolute values of x is

$$|-4| + |3| = 4 + 3 = 7$$

Note that the question is asking for the sum of the absolute values of x not the absolute value of the sum.

12. 3

Equation $|2x + 1| = 3$

Solve for the positive value:

$$2x + 1 = 3 \rightarrow 2x = 3 - 1 \rightarrow 2x = 2 \rightarrow x = 1$$

It is given $x > 0$. The above value satisfies the condition. Hence, no need to solve for the other value.

Equation $|2y + 3| = 7$

Solve for the positive value:

$$2y + 3 = 7 \rightarrow 2y = 7 - 3 \rightarrow 2y = 4 \rightarrow y = 2$$

It is given $y > x$. Since $2 > 1$, the above value of y satisfies the condition. Hence, no need to solve for the other value.

Check for an extraneous solution: 1 and 2 are not extraneous solutions.

Solve for $x + y$:

$$1 + 2 = 3$$

(Note that when the negative values for x and y are calculated, they do not satisfy $y > x > 0$. Hence, 3 is the only possible answer.)

Section 8 – Ratios, Proportions, and Rates

Category 42 – Ratios and Proportions

1. D

Set up a proportion:

Let the number of students in music class $= x$.

$$\frac{\text{students in music class}}{\text{students in drama class}} = \frac{2}{5} = \frac{x}{120}$$

$$5x = 2 \times 120 \;\rightarrow\; 5x = 240 \;\rightarrow\; x = 48$$

Determine total number of students: Since each student must select one of the classes, the total number of students is the sum of the number of students in both the classes.

$$120 + 48 = 168$$

2. C

Set up a proportion:

Total number of trays $= 4 + 7 = 11$.

Total amount of butter $= 5\frac{1}{2} = 5.5$ cups.

Let the cups of butter for 4 trays $= x$.

$$\frac{\text{number of trays}}{\text{cups of butter}} = \frac{11}{5.5} = \frac{4}{x}$$

$$11x = 4 \times 5.5 \;\rightarrow\; 11x = 22 \;\rightarrow\; x = 2$$

3. D

Equate the ratios: Since the ratios are equivalent, they can be equated.

Since 24 is double of 12, $2(a{:}\,5{:}\,12) = 3{:}\,b{:}\,24$. Hence,

$$2a{:}\,10{:}\,24 = 3{:}\,b{:}\,24$$

$$2a = 3 \;\rightarrow\; a = \frac{3}{2}$$

$$b = 10$$

Determine $2(a + b)$:

$$2\left(\frac{3}{2} + 10\right) = 2 \times 11.5 = 23$$

4. A

Note that if $2{:}\,3$ is multiplied by 10, the ratio of muffins will be $20{:}\,30$. This gives a total of 50 muffins with the difference $= 30 - 20 = 10$.

Alternatively, proportion can be set up to determine the number of muffins and, subsequently, the difference.

5. 1.4

Since 12 bottles cost $16.80, one bottle will cost

$$\frac{16.80}{12} = 1.40$$

6. 210

Set up a proportion:

Let the distance between airports in miles $= x$.

$$\frac{\text{inches}}{\text{miles}} = \frac{0.5}{14} = \frac{7.5}{x}$$

$$0.5x = 7.5 \times 14 \;\rightarrow\; 0.5x = 105 \;\rightarrow\; x = 210$$

7. 90.7

Set up a proportion:

Let the weight in kilograms $= x$.

$$\frac{\text{pounds}}{\text{kilograms}} = \frac{2.20462}{1} = \frac{200}{x}$$

$$2.20462x = 200 \times 1 \;\rightarrow\; x = 90.718 = 90.7$$

8. C

Protein mix A:

Protein in 1 cup $= 28$ grams. Hence, protein in $\frac{1}{4}$ cup is

$$28 \times \frac{1}{4} = 7$$

Protein mix B:

Protein in 1 cup $= 20$ grams. Hence, protein in $\frac{3}{4}$ cup is

$$20 \times \frac{3}{4} = 15$$

Determine total protein in 1 cup mixture Laura created:

$$7 + 15 = 22$$

9. C

Determine the ratio as a fraction:

Product A at both companies $= 1{,}812$.

Product A + Product B at both companies $= 3{,}020$.

$$\frac{1{,}812}{3{,}020} = 0.6 = \frac{6}{10} = \frac{3}{5}$$

10. D

Since the length is increased, answer choices A and C can be eliminated.

Set up a proportion and solve:

$GH = 21$. $FG = 6$.

Increased $FG = 6 + 5 = 11$.

Let the increase in units of $GH = x$.

Hence, increased $GH = 21 + x$.

$$\frac{FG}{GH} = \frac{\text{Increased } FG}{\text{Increased } GH} \;\rightarrow\; \frac{6}{21} = \frac{11}{21 + x} \;\rightarrow$$

$$6 \times (21 + x) = 11 \times 21 \;\rightarrow\; 126 + 6x = 231 \;\rightarrow$$

$$6x = 231 - 126 = 105 \;\rightarrow\; x = 17.5$$

11. B

Regular sugar cookies: Since each packet contains 12 ounces, 48 ounces will be in $\frac{48}{12} = 4$ packets.

Each packet contains 8 cookies. Hence,

$R =$ total cookies in 48 ounces $= 4 \times 8 = 32$

Low sugar cookies: Since each packet contains 8 ounces, 48 ounces will be in $\frac{48}{8} = 6$ packets.

Each packet contains 12 cookies. Hence,

$L =$ total cookies in 48 ounces $= 6 \times 12 = 72$

Determine the difference:

$$L - R = 72 - 32 = 40$$

12. C

Set up a proportion for each floor:

$$\frac{\text{Number of rooms cleaned}}{\text{Number of housekeepers}} = \frac{18}{1} = \frac{x}{K} \rightarrow$$

$$18K = x \rightarrow K = \frac{x}{18}$$

Determine K for 3 floors:

For 3 floors K will be 3 times. Hence,

$$K = \frac{x}{18} \times 3 = \frac{x}{6}$$

Category 43 – Rates

1. B

Use the conversion factors: 1 hour = 60 minutes.

$$10 \text{ ~~minutes~~} \times \frac{30 \text{ miles}}{60 \text{ ~~minutes~~}} = 5 \text{ miles}$$

2. D

Use the conversion factors: 1 minute = 60 seconds.

$$y \text{ ~~inches~~} \times \frac{60 \text{ seconds}}{x \text{ ~~inches~~}} = \frac{60y}{x} \text{ seconds}$$

3. 105

Use the conversion factors: 1 hour = 60 minutes.

$$140 \text{ ~~minutes~~} \times \frac{45 \text{ miles}}{60 \text{ ~~minutes~~}} = 105 \text{ miles}$$

4. 12

4 miles = one-fifth of 20 miles. Hence, it will take her one-fifth the time = 12 minutes. See below calculation using conversion factors.

$$4 \text{ miles} \times \frac{60 \text{ minutes}}{20 \text{ miles}} = 12 \text{ minutes}$$

5. 12

Use the conversion factors:

$$240 \text{ ~~packets~~} \times \frac{1 \text{ minute}}{20 \text{ ~~packets~~}} = 12 \text{ minutes}$$

6. A

Use the conversion factors: 1 minute = 60 seconds.

$$\frac{768 \text{ ~~ounces~~}}{60 \text{ seconds}} \times \frac{1 \text{ gallon}}{128 \text{ ~~ounces~~}} =$$

0.1 gallon per second

7. D

Use the conversion factors: 1 hour = 60 minutes.

Jolie: 10 miles in 60 minutes.

$$15 \text{ ~~minutes~~} \times \frac{10 \text{ miles}}{60 \text{ ~~minutes~~}} = 2.5 \text{ miles}$$

Charlie: 18 miles in 60 minutes.

$$15 \text{ ~~minutes~~} \times \frac{18 \text{ miles}}{60 \text{ ~~minutes~~}} = 4.5 \text{ miles}$$

Determine the distance between Jolie and Charlie: They are traveling in the opposite direction.

$$2.5 + 4.5 = 7 \text{ miles}$$

8. D

Total miles = 3,459.

At the average speed of 526 miles per hour, miles traveled in 4 hours are $526 \times 4 = 2{,}104$.

The remaining miles traveled for x minutes at 420 miles per hour are $3{,}459 - 2{,}104 = 1{,}355$ miles.

Use the conversion factors to determine x: 1 hour = 60 minutes.

$$1{,}355 \text{ ~~miles~~} \times \frac{60 \text{ minutes}}{420 \text{ ~~miles~~}} = 193.57 \text{ minutes} = 194$$

9. A

Use the conversion factors: 1 hour = 60 minutes.

Bird A: 18 miles in 60 minutes.

$$10 \text{ ~~minutes~~} \times \frac{18 \text{ miles}}{60 \text{ ~~minutes~~}} = 3 \text{ miles}$$

Bird B: 24 miles in 60 minutes.

$$10 \text{ ~~minutes~~} \times \frac{24 \text{ miles}}{60 \text{ ~~minutes~~}} = 4 \text{ miles}$$

Determine the distance between the two birds: The birds are travelling in the same direction.

$$4 - 3 = 1 \text{ mile}$$

10. C

Use the conversion factors: 1 hour = 3,600 seconds.

$$\frac{36 \text{ }\cancel{\text{gallons}}}{3{,}600 \text{ seconds}} \times \frac{128 \text{ ounces}}{1 \text{ }\cancel{\text{gallon}}} =$$
$$1.28 \text{ ounces per second}$$

11. B

Use the conversion factors: 1 hour = 60 minutes.

First 15 minutes: 60 miles in 60 minutes.

$$15 \text{ }\cancel{\text{minutes}} \times \frac{60 \text{ miles}}{60 \text{ }\cancel{\text{minutes}}} = 15 \text{ miles}$$

Remaining 30 minutes: 40 miles in 60 minutes.

$$30 \text{ }\cancel{\text{minutes}} \times \frac{40 \text{ miles}}{60 \text{ }\cancel{\text{minutes}}} = 20 \text{ miles}$$

Determine the total miles traveled:

$$15 + 20 = 35 \text{ miles}$$

12. 50

Use the conversion factors: 1 hour = 3600 seconds.

$$\frac{180 \text{ }\cancel{\text{kilometers}}}{3{,}600 \text{ seconds}} \times \frac{1000 \text{ meters}}{1 \text{ }\cancel{\text{kilometer}}} =$$
$$50 \text{ meters per second}$$

13. 50.7

John travels 100 miles per day. In 4 days, John will travel

$$4 \times 100 = 400 \text{ miles}$$

Use the conversion factors:

$$400 \text{ }\cancel{\text{miles}} \times \frac{1 \text{ }\cancel{\text{gallon}}}{30 \text{ }\cancel{\text{miles}}} \times \frac{3.8 \text{ liters}}{1 \text{ }\cancel{\text{gallon}}} =$$
$$50.66 \text{ liters} = 50.7 \text{ liters}$$

14. C

Use the conversion factors:

$$2 \text{ }\cancel{\text{hours}} \times \frac{50 \text{ }\cancel{\text{miles}}}{1 \text{ }\cancel{\text{hour}}} \times \frac{1 \text{ }\cancel{\text{gallon}}}{20 \text{ }\cancel{\text{miles}}} \times \frac{3 \text{ dollars}}{1 \text{ }\cancel{\text{gallon}}} =$$
$$15 \text{ dollars}$$

Section 8 – Drill

1. B

10 kilometers is one-third of 30 kilometers. Hence, it will take her one-third the time = 20 minutes.

2. D

Use the conversion factors:

$$3 \text{ hours} = 3 \times 60 = 180 \text{ minutes}$$
$$180 \text{ }\cancel{\text{minutes}} \times \frac{p \text{ pages}}{m \text{ }\cancel{\text{minutes}}} = \frac{180p}{m} \text{ pages}$$

3. C

Use the conversion factors: 1 hour = 3,600 seconds.

$$0.2 \text{ }\cancel{\text{miles}} \times \frac{3{,}600 \text{ seconds}}{60 \text{ }\cancel{\text{miles}}} = 12 \text{ seconds}$$

4. C

Evaluate each answer choice:

$$\text{salt:water} = 20{:}500 = 1{:}25$$

Since each proportion will have the same ratio, evaluate the above ratio against the ratio in each answer choice.

Choice I: $1{:}25$. This matches the given ratio.

Choice II: $3{:}50 = 1{:}16.7$. This does not match the given ratio.

Choice III: $5{:}125 = 1{:}25$. This matches the given ratio.

5. A

Set up a proportion:

$$\frac{\text{annual rainfall}}{\text{March rainfall}} \rightarrow \frac{12.4}{3.1} = \frac{r}{\text{March rainfall}}$$
$$\text{March rainfall} = \frac{3.1r}{12.4} = \frac{r}{4}$$

6. D

Use the conversion factors: 1 hour = 3,600 seconds.

$$\frac{120 \text{ }\cancel{\text{miles}}}{3{,}600 \text{ seconds}} \times \frac{5{,}280 \text{ feet}}{1 \text{ }\cancel{\text{mile}}} = 176 \text{ feet per second}$$

7. A

Use the conversion factors:

$$3{,}280 \text{ }\cancel{\text{feet}} \times \frac{1 \text{ miles}}{5{,}280 \text{ }\cancel{\text{feet}}} = 0.62 \text{ miles}$$

The question asks for an approximate answer. Answer choice A is closest.

8. 6

1 mile = 0.2 centimeters. Hence,

$$30 \text{ miles} = 0.2 \times 30 = 6 \text{ centimeters}$$

9. <u>50</u>

Plane A is flying at the speed of 600 miles per 60 minutes. In 30 minutes, the distance will be half = 300 miles.

Plane B is flying at the speed of 500 miles per 60 minutes. In 30 minutes, the distance will be half = 250 miles.

The difference in miles $= 300 - 250 = 50$.

(See below calculation for miles in 30 minutes using conversion factors.

Plane A: 600 miles in 60 minutes.

$$30\ \cancel{\text{minutes}} \times \frac{600\text{ miles}}{60\ \cancel{\text{minutes}}} = 300\text{ miles}$$

Plane B: 500 miles in 60 minutes.

$$30\ \cancel{\text{minutes}} \times \frac{500\text{ miles}}{60\ \cancel{\text{minutes}}} = 250\text{ miles}$$

)

10. <u>32</u>

Use the conversion factor:

$$1{,}400\ \cancel{\text{dollars}} \times \frac{100\ \cancel{\text{square feet}}}{250\ \cancel{\text{dollars}}} \times \frac{8\text{ hours}}{140\ \cancel{\text{square feet}}} = 32\text{ hours}$$

11. <u>1.5</u>

Convert the centimeter scale to miles: For ease, convert the scale to 1 centimeter. Multiply both numbers by 2.

$$\frac{1}{2} \times 2\text{ centimeters} = 24\text{ miles} \times 2 \rightarrow$$

$$1\text{ centimeter} = 48\text{ miles}$$

Multiply the distance given in centimeters by 48 to convert to miles. See table below.

Names of roads and highways	Distance in centimeter	Distance in miles
State Road 27	$\frac{1}{8}$	$\frac{1}{8} \times 48 = 6$
Highway 2	$1\frac{1}{4}$	$\frac{5}{4} \times 48 = 60$
Highway 17	$\frac{1}{4}$	$\frac{1}{4} \times 48 = 12$
State Road 4	$\frac{3}{4}$	$\frac{3}{4} \times 48 = 36$

Use the conversion factor:

Total distance on State Road 27 and State Road 4 is

$$6 + 36 = 42\text{ miles}$$

$$42\ \cancel{\text{miles}} \times \frac{1\text{ gallon}}{28\ \cancel{\text{miles}}} = 1.5\text{ gallons}$$

12. <u>D</u>

Determine the total number of cookies and profit:

Let the proportion $= x$. Hence,

$$3x + 4x + 5x = 36 \rightarrow 12x = 36 \rightarrow x = 3$$

Sugar cookies:

Number $= 3 \times 3 = 9$. Profit $= 9 \times 0.52 = 4.68$

Oatmeal cookies:

Number $= 4 \times 3 = 12$. Profit $= 12 \times 0.42 = 5.04$

Chocolate cookies:

Number $= 5 \times 3 = 15$. Profit $= 15 \times 0.32 = 4.8$

Total profit for the tray $= 4.68 + 5.04 + 4.8 = 14.52$

Section 9 – Percentages

Category 44 – Percentages of a Number and Percent Increase/Decrease

1. 744

Convert percent to decimal: $6\% = 0.06$

Determine the number of residents: Total $= 12{,}400$.

$$12{,}400 \times 0.06 = 744$$

2. D

Convert percent to decimal:

$$\text{apple} + \text{pear} + \text{other} = 0.38 + 0.15 + 0.03 = 0.56$$

Determine percent: Total $= 425$.

$$425 \times 0.56 = 238$$

3. 9

Convert percent to decimal:

$$25\% = 0.25.\ 30\% = 0.3.\ 200\% = 2.0$$

Determine the final value of 60:

$$0.25 \times 0.3 \times 2 \times 60 = 9$$

4. A

Convert percent to decimal:

$$6\% \text{ more} = 1 + 1.06 = 1.06$$

Determine the 2019 bonus: Estimated bonus $= d$.

$$1.06 \times d = 1.06d$$

5. C

Convert percent to decimal: $20\% = 0.2$

Determine the number of CDs to brother: starting $= 125$.

$$x = 0.2 \times 125 = 25$$

Determine the number of CDs to cousin: starting $=$ $125 - 25 = 100$.

$$y = 0.2 \times 100 = 20$$

Hence, $x > y$.

6. A

Convert percent to decimal:

$$35\% \text{ discount} = 35\% \text{ decrease} = 1 - 0.35 = 0.65$$

$$12\% \text{ discount} = 12\% \text{ decrease} = 1 - 0.12 = 0.88$$

$$8\% \text{ sales tax} = 8\% \text{ increase} = 1 + 0.08 = 1.08$$

Determine the final value of p:

$$(0.65)(0.88)(1.08)(p)$$

7. 80

Convert percent to decimal:

$$20\% \text{ decrease} = 1 - 0.2 = 0.8$$

$$25\% \text{ increase} = 1 + 0.25 = 1.25$$

Determine the history grade of the 3rd test:

$$80 \times 0.8 \times 1.25 = 80$$

8. 957

Convert percent to decimal:

$$20\% \text{ checked out} = 20\% \text{ decrease} = 1 - 0.2 = 0.8$$

Determine the remaining books:

$$0.8 \times 1{,}040 = 832$$

Determine total number of books at the end of the day:

$$\text{remaining} + \text{return} = 832 + 125 = 957$$

9. C

Convert percent to decimal:

$$160\% \text{ increase} = 1 + 1.6 = 2.6$$

Determine the final number of residents by 2020:

$$2.6 \times 3{,}680 = 9{,}568$$

Additional residents $= 9{,}568 - 3{,}680 = 5{,}888$

10. C

Convert percent to decimal:

45% less $= 45\%$ decrease $= 1 - 0.45 = 0.55$

320% more $= 320\%$ increase $= 1 + 3.2 = 4.2$

Determine the value of a: $b = 4.2 \times 6 = 25.2$.

$$a = 0.55b = 0.55 \times 25.2 = 13.86$$

11. 25

Determine the percent of each balloon:

Let gold balloons $= x$.

Let silver balloons $= y$.

Let white balloons $= z$.

$$\text{Silver} = y = 20\% \text{ less than white} = 1 - 0.2z = 0.8z$$

$$\text{gold} = x = 25\% \text{ greater than silver} = 1.25y = 1.25(0.8z)$$

Determine number of white balloons: Total $= 70$.

$$1.25(0.8z) + 0.8z + z = 70 \ \rightarrow\ z + 0.8z + z = 70 \ \rightarrow\ 2.8z = 70 \ \rightarrow\ z = 25$$

12. C

Convert percent to decimal:

$$\text{first month } 12.5\% \text{ withdrawal} = 0.125$$

$$\text{second month } 10\% \text{ greater than } 0.125 = 0.125 \times 1.1 = 0.1375$$

Hence,

$$\text{total withdrawals} = 0.125 + (0.125 \times 1.1) = 0.2625$$

Determine the final number: Starting $= 4{,}000$.

$$\text{Total withdrawals} = 4{,}000 \times 0.2625 = 1{,}050$$

$$\text{remaining balance} = 4{,}000 - 1{,}050 = 2{,}950$$

13. D

Convert percent to decimal:

$$\text{white sedans with leather seats} =$$
$$18\% \text{ of } 32\% \text{ of } 52\% =$$
$$0.18 \times 0.32 \times 0.52 = 0.029952$$

Determine percent of white sedans with leather seats:

$$0.029952 \times 100 = 2.995\% = 3\% \text{ approximately}$$

Determine percent of cars NOT white sedans with leather seats:

$$100\% - 3\% = 97\%$$

14. 7.5

Determine the equation:

$$x \text{ ml of } 4\% \text{ solution} + 2 \text{ ml of } 80\% \text{ solution} = (x+2) \text{ ml of } 20\% \text{ solution}$$

Convert percent solution to amount:

$$2 \text{ ml of } 80\% \text{ solution} = 2 \times 0.8 = 1.6$$
$$x \text{ ml of } 4\% \text{ solution} = 0.04x$$
$$(x+2) \text{ ml of } 20\% \text{ solution} = (x+2)0.2 = 0.2x + 0.4$$

Solve: Plug the amounts into the equation.

$$1.6 + 0.04x = 0.2x + 0.4 \rightarrow$$
$$1.6 - 0.4 = 0.2x - 0.04x \rightarrow 1.2 = 0.16x \rightarrow x = 7.5$$

Category 45 – The Original Number before a Percent Increase/Decrease

1. A

Convert percent to decimal:

$$20\% \text{ decrease} = 1 - 0.2 = 0.8$$
$$5\% \text{ increase} = 1 + 0.05 = 1.05$$

Determine number n: End number = 42.

$$\frac{42}{(0.8)(1.05)}$$

2. D

Convert percent to decimal:

$$27\% \text{ stores close} = 27\% \text{ decrease} = 1 - 0.27 = 0.73$$

Determine the number of stores in 2018: End number of stores in 2022 = 584.

$$\frac{584}{0.73} = 800$$

3. C

Convert percent to decimal:

$$30\% \text{ discount} = 30\% \text{ decrease} = 0.7$$
$$5\% \text{ discount} = 5\% \text{ decrease} = 0.95$$

Determine the original price of book: End price = \$5.32.

$$\frac{5.32}{0.7 \times 0.95} = 8$$

4. B

Convert percent to decimal:

$$20\% \text{ increase} = 1 + 0.2 = 1.2$$
$$10\% \text{ decrease} = 1 - 0.1 = 0.9$$

Determine number p: End value = 378.

$$\frac{378}{1.2 \times 0.9} = 350$$

5. C

Convert percent to decimal:

$$40\% \text{ discount} = 40\% \text{ decrease} = 1 - 0.4 = 0.6$$
$$6\% \text{ sales tax} = 6\% \text{ increase} = 1 + 0.06 = 1.06$$

Determine the original price of shirt: End price = \$15.90.

$$\frac{15.90}{0.6 \times 1.06} = 25$$

6. A

Convert percent to decimal:

$$25\% \text{ increase from 2017 to 2018} = 1 + 0.25 = 1.25$$
$$20\% \text{ increase from 2016 to 2017} = 1 + 0.2 = 1.2$$

Determine the 2016 profit: End profit in 2018 = 10.5.

$$\frac{10.5}{1.2 \times 1.25} = 7$$

Category 46 – A Number Percent of Another Number

1. B

Determine 12 is what percent of 30:

$$\frac{12}{30} \times 100 = 40$$

2. 1075

Determine 84% of what number is 903:

Let the number = y.

$$\frac{903}{y} \times 100 = 84 \rightarrow 84y = 90{,}300 \rightarrow$$
$$y = \frac{90{,}300}{84} = 1{,}075$$

3. D

Determine 3.20 is what percent of 40 (initial pass value):

$$\frac{3.20}{40} \times 100 = 8\%$$

4. D

Determine the increased value of p:

$$25\% \text{ increase} = 1 + 0.25 = 1.25$$
$$\text{increased value of } p = 1.25p$$

Determine p is what percent of $1.25p$:

$$\frac{p}{1.25p} \times 100 = 80\%$$

5. <u>A</u>

<u>Percent discount at Store A</u>

$$\text{Discount} = 250 - 215 = \$35$$

Determine 35 is what percent of 250:

$$\frac{35}{250} \times 100 = 14\%$$

<u>Percent discount at Store B</u>

$$\text{Discount} = 250 - 200 = \$50$$

Determine 50 is what percent of 250:

$$\frac{50}{250} \times 100 = 20\%$$

Determine the difference: $20\% - 14\% = 6\%$

6. <u>B</u>

Determine the price of computer at Store B = a:

$$10\% \text{ additional discount} = 1 - 0.1 = 0.9$$

$$a = 1{,}180 \times 0.9 = \$1{,}062$$

Determine $a = 1{,}062$ is what percent of 1,200:

$$\frac{1{,}062}{1{,}200} \times 100 = 88.5\%$$

7. <u>C</u>

Determine the original price without sales tax:

$$\frac{43.20}{1.08} = \$40$$

Sales tax = $43.20 - 40 = \$3.20$.

Determine the refunded price without sales tax:

$$36 - 3.20 = \$32.80$$

Determine 32.80 is what percent of 40:

$$\frac{32.80}{40} \times 100 = 82\%$$

Category 47 – Percent Change

1. <u>B</u>

Determine the percent change:

Old profit = 1.2. New profit = 1.6.

$$\frac{1.6 - 1.2}{1.2} \times 100 = \frac{0.4}{1.2} \times 100 = 33.33\%$$

2. <u>C</u>

Determine the percent change:

Old price = 20. New price = 8.

$$\frac{8 - 20}{20} \times 100 = -\frac{12}{20} \times 100 = -60\%$$

3. <u>B</u>

Determine the percent change: Remove the warranty from the original and reduced prices.

Old price = $3{,}200 - 200 = \$3{,}000$.

Reduced price = new price = $2{,}300 - 200 = \$2{,}100$.

$$\frac{2{,}100 - 3{,}000}{3{,}000} \times 100 = -\frac{900}{3{,}000} \times 100 = -30\%$$

4. <u>B</u>

Determine the percent change: It is apparent that the change in answer choices A, C, and D is lower than in answer choice B.

If unsure, determine the percent change for each answer choice.

5. <u>150</u>

Determine the percent change:

March = 10,000. May = 25,000.

$$p = \frac{\text{households in May} - \text{households in March}}{\text{households in March}} \times 100$$

$$= \frac{25{,}000 - 10{,}000}{10{,}000} \times 100 = \frac{15{,}000}{10{,}000} \times 100 = 150$$

Section 9 – Drill

1. <u>C</u>

Convert percent to decimal: $14\% = 0.14$

Determine the final value of 1,050:

$$0.14 \times 1{,}050 = 147$$

2. <u>132</u>

Convert percent to decimal: $48\% = 0.48$

Determine the number of pear trees: Total = 275.

$$0.48 \times 275 = 132$$

3. <u>140</u>

Convert percent to decimal:

$$20\% \text{ stores close} = 20\% \text{ decrease} = 1 - 0.2 = 0.8$$

Determine the initial number of stores in 2015: End number of stores in 2018 = 112.

$$\frac{112}{0.8} = 140$$

4. A

Convert percent to decimal:

$$15\% \text{ discount} = 15\% \text{ decrease} = 1 - 0.15 = 0.85$$

$$8\% \text{ sales tax} = 8\% \text{ increase} = 1 + 0.08 = 1.08$$

Determine the original price of the scarf: End price $= d$.

$$\frac{d}{(0.85)(1.08)}$$

5. C

Remaining balance $= 75 - 9 - 17.25 = \$48.75$.

Determine 48.75 (remaining balance) is what percent of 75 (initial value):

$$\frac{48.75}{75} \times 100 = 65\%$$

6. D

Calculate the percent change: Old = 12. New = 14.

$$\frac{14 - 12}{12} \times 100 = \frac{2}{12} \times 100 = 16.67\%$$

Since the change is positive, it is an increase by 16.67%.

7. B

Convert percent to decimal:

$$6\% \text{ decrease} = 1 - 0.06 = 0.94$$

$$19\% \text{ increase} = 1 + 0.19 = 1.19$$

$$22\% \text{ increase} = 1 + 0.22 = 1.22$$

Determine the final value: Initial population $= s$.

$$(0.94)(1.19)(1.22)(s)$$

8. D

Determine the remaining amount on cashless card:

Since 36 is 12 times of 3, the cost of 36 rounds is

$$0.5 \times 12 = 6$$

Hence, \$6 were deducted for 36 rounds of games.

Amount remaining on cashless card $= 20 - 6 = \$14$.

Determine the remaining amount $= 14$ is what percent of 20:

$$\frac{14}{20} \times 100 = 70\%$$

9. B

Convert percent to decimal:

$$25\% \text{ decrease} = 1 - 0.25 = 0.75$$

$$25\% \text{ increase} = 1 + 0.25 = 1.25$$

Determine the final value of 80: $80 \times 1.25 \times 0.75 = 75$.

Determine the percent change: Old = 80. New = 75.

$$\frac{75 - 80}{80} \times 100 = -\frac{5}{80} \times 100 = -6.25\%$$

10. 480

Determine the percent difference:

Students in cooking club $= 40\%$.

Students not in cooking club $= 100\% - 40\% = 60\%$.

Hence, $60\% - 40\% = 20\%$ less students joined the cooking club.

It is given that 96 less students joined the cooking club. Hence, 96 is 20% of the students.

Determine the total number of students:

Since 20% of students $= 96$,

$$100\% \text{ of students} = 96 \times 5 = 480$$

11. 350

Convert percent to decimal: $40\% = 0.4$

Determine the total members enrolled in the club:

Members in yoga and karate class $= 140$. Hence, total is

$$\frac{140}{0.4} = 350$$

12. 62.5

Determine the equation:

10 liters of 25% solution $+$ 2 liters of $p\%$ solution $=$
$(2 + 10)$ liters of 31.25% solution

Convert percent solution to amount:

$$10 \text{ liters of } 25\% \text{ solution} = 10 \times 0.25 = 2.5$$

$$2 \text{ liters of } p\% \text{ solution} = 2 \times \frac{p}{100} = \frac{p}{50}$$

$$(10 + 2) \text{ liters of } 31.25\% \text{ solution} = 0.3125(10 + 2) = $$
$$= 0.3125(12) = 3.75$$

Solve: Plug the amounts into the equation.

$$2.5 + \frac{p}{50} = 3.75 \ \rightarrow \ \frac{p}{50} = 3.75 - 2.5 \ \rightarrow$$

$$\frac{p}{50} = 1.25 \ \rightarrow \ p = 1.25 \times 50 = 62.5$$

13. 30

Determine x: $n = 40$.

$$10\% \text{ decrease} = 1 - 0.1 = 0.9$$

$$x = 0.9 \times 40 = 36$$

Determine p: $x = 36$.

$$20\% \text{ increase} = 1 + 0.2 = 1.2$$

$$p = \frac{36}{1.2} = 30$$

14. B

End population of Bird A in 2019.

$$5\% \text{ increase from 2016 to 2019} = 1 + 0.05 = 1.05$$

Determine the end population in 2019:

$$6{,}000 \times 1.05 = 6{,}300$$

Initial population of Bird B in 2016.

$$20\% \text{ increase from 2016 to 2019} = 1 + 0.2 = 1.2$$

Determine the initial population in 2016: End population in 2019 $= 6{,}300$ from end population of Bird A.

$$\frac{6{,}300}{1.2} = 5{,}250$$

Section 10 – Exponents and Exponential Functions

Category 48 – Exponents

1. C

Make the bases same: 27 can be written as (3^3).

$$3^2 \times (3^3)^4 = 3^2 \times 3^{3\times 4} = 3^{2+12} = 3^{14}$$

2. B

Plug in $x = 0$ and solve:

$$y = 259(a)^0 \rightarrow y = 259(1) \rightarrow y = 259$$

3. C

$$(3y)^{\frac{3}{a}} = \sqrt[a]{(3y)^3} = \sqrt[a]{27y^3}$$

4. 5

Since 64 and m^3 are perfect cubes, the cube root can be removed from the left expression.

$$\sqrt[3]{4^3m^3} = 20 \rightarrow \sqrt[3]{(4m)^3} = 20 \rightarrow 4m = 20 \rightarrow m = 5$$

5. A

Replace roots with fractional exponents and simplify:

$$r^{\frac{1}{2}} \times r^{\frac{1}{3}} = r^{\frac{1}{2}+\frac{1}{3}} = r^{\frac{3+2}{6}} = r^{\frac{5}{6}}$$

6. C

Replace roots with fractional exponent and simplify:

$$\frac{5^{\frac{1}{2}} \times a^{\frac{4}{2}}}{5^{\frac{1}{4}} \times a^{\frac{3}{4}}} = \frac{5^{\frac{1}{2}} \times a^2}{5^{\frac{1}{4}} \times a^{\frac{3}{4}}}$$

Move expressions in the denominator to the numerator:

$$5^{\frac{1}{2}-\frac{1}{4}} \times a^{2-\frac{3}{4}} = 5^{\frac{2-1}{4}} \times a^{\frac{8-3}{4}} = 5^{\frac{1}{4}} \times a^{\frac{5}{4}} = \sqrt[4]{5a^5}$$

7. B

Simplify: 27 can be written as 3^3.

$$(3^3 \times a^9)^{\frac{1}{3}} = 3^{3\times\frac{1}{3}} \times a^{9\times\frac{1}{3}} = 3a^3$$

8. A

Make the bases same: All bases are multiples of 3.

$$((3^2)^3)^{n+1} = (3^4)^3 \times 3^{3n} \rightarrow 3^{6(n+1)} = 3^{12+3n}$$

Equate the exponents:

$$6(n+1) = 12 + 3n \rightarrow 6n + 6 = 12 + 3n \rightarrow 3n = 6 \rightarrow n = 2$$

9. B

Make the bases same: All bases are multiples of 2.

$$(2^2)^{3a-1} = 2^{3+a} \times (2^5)^b \rightarrow 2^{2(3a-1)} = 2^{3+a+5b}$$

Equate the exponents:

$$2(3a-1) = 3 + a + 5b \rightarrow 6a - 2 = 3 + a + 5b \rightarrow 6a - a - 5b = 3 + 2 \rightarrow a - b = 1 \rightarrow b = a - 1$$

10. D

Remove perfect squares and write as square root:

$$(4^2x^4xy^2y)^{\frac{1}{2}} = 4x^2y(xy)^{\frac{1}{2}} = 4x^2y\sqrt{xy}$$

11. A

Make the bases same:

$$3^{2(4a)} = 3^{\frac{4}{5}} \rightarrow 3^{8a} = 3^{\frac{4}{5}}$$

Equate the exponents and solve for a:

$$8a = \frac{4}{5} \rightarrow a = \frac{4}{5 \times 8} = \frac{1}{10}$$

12. D

If the expression 2^ay^4 is divided by the expression 2^by^4, then y^4 will cancel out. 2^a divided by 2^b is 2^{a-b}.

$$\frac{2^a\cancel{y^4}}{2^b\cancel{y^4}} = \frac{80}{5} \rightarrow 2^{a-b} = 16 \rightarrow 2^{a-b} = 2^4$$

Equate the exponents:

$$a - b = 4$$

13. A

Move the expression in the denominator to the numerator:

$$(x^5y^{-\frac{1}{2}}) \times (x^{\frac{1}{2}}y^{-3}) \times (x^{-\frac{9}{2}}y^{\frac{9}{2}})$$

Add exponents of similar bases:

$$x^5x^{\frac{1}{2}}x^{-\frac{9}{2}}y^{-\frac{1}{2}}y^{-3}y^{\frac{9}{2}} = x^{5+\frac{1}{2}-\frac{9}{2}}y^{-\frac{1}{2}-3+\frac{9}{2}} = xy$$

14. 3.6 or 18/5

Replace roots with fractional exponents and move the expression in the denominator to the numerator:

$$\frac{\sqrt{s^{3+5}}}{\sqrt[5]{s^2}} = s^{\frac{x}{y}} \rightarrow s^{\frac{8}{2}} \times s^{-\frac{2}{5}} = s^{\frac{x}{y}} \rightarrow s^{4-\frac{2}{5}} = s^{\frac{x}{y}}$$

Equate the exponents:

$$\frac{x}{y} = 4 - \frac{2}{5} = \frac{20-2}{5} = \frac{18}{5} = 3.6$$

15. 20

$$a^6 = a^2 \times a^2 \times a^2 = 10 \times 10 \times 10 = 1{,}000$$

$$b^{-5} = \frac{1}{b^5} = \frac{1}{50}$$

$$(a^6)(b^{-5}) = (1{,}000)\left(\frac{1}{50}\right) = 20$$

Category 49 – Linear Versus Exponential Growth and Decay

1. B

In tables A, C, and D the change in the value of y with the change in the value of x is constant. In table B, y increases 4-fold for every 2 increase in the value x. This is exponential increase/growth.

2. D

Since the colonies double every 30 minutes, there are twice more colonies of bacteria every 30 minutes.

3. A

Since the decrease is the same each year, it is a linear decrease.

4. B

Since the population is projected to grow exponentially each year, it will increase by a certain percent each year than in the preceding year.

5. C

Since Team A is ahead of Team B by one-fourth the distance every 20 minutes than the preceding 20 minutes, the distance will decrease every 20 minutes.

Category 50 – Exponential Growth and Decay

1. C

Determine the components of exponential growth:

$a = 3.2$. Since a is the initial number, 3.2 is the initial population in 1990.

2. C

Determine the components of exponential growth:

$r = 9\% = 0.09$. $c = b = 1 + r = 1 + 0.09 = 1.09$.

3. A

Determine the components of exponential decay:

$a = 200{,}000$.

$r = 50\% = 0.5$. $x = 1 - r = 1 - 0.5 = 0.5$.

Time interval $=$ 10 minutes. In 30 minutes, there will be $\frac{30}{10} = 3$ time intervals. Hence, $t = 3$.

$$y = a(b)^x = 200{,}000(0.5)^3 = 25{,}000$$

4. A

Determine the components of exponential growth:

$a = \$12$.

$f(m) =$ price in m months after May 2020. Hence, $f(3) =$ price in 3 months after May 2020.

Since $f(3) = 22$, it is the price in 3 months after May 2020.

5. D

Determine the components of exponential growth:

$b = 1 + r = 1.16 = 1 + 0.16$. Hence, $r = 0.16 = 16\%$.

Since time interval $= \frac{t}{2}$, the increase is every 2 years.

Hence, every 2 years P increases by 16%.

This eliminates answer choices A, B, and C.

6. D

Determine the components of exponential decay:

$a = 150$.

$r = 1\% = 0.01$. $b = 1 - r = 1 - 0.01 = 0.99$.

Time interval is every 4 months $= \frac{m}{4}$.

$D =$ decay after m months.

$$y = a(b)^x \;\rightarrow\; D = 150(0.99)^{\frac{m}{4}}$$

7. B

Determine the components of exponential growth:

$(1.26)^{\frac{n}{3}}$ is same as $(1.26)^{\frac{1}{3}n} = \left((1.26)^{\frac{1}{3}}\right)^n = (1.08)^n$ approximately.

The equation becomes $G(n) = 2{,}300(1.08)^n$.

$b = 1 + r = 1.08 = 1 + 0.08 = 1 + \frac{8}{100}$. Hence,

$$G(n) = 2{,}300\left(1 + \frac{8}{100}\right)^n \;\rightarrow\; k = 8$$

8. B

Determine the components of compound interest formula:

$P = 2{,}000$. $r = 8\% = 0.08$. $t = 3$.

Since the interest is compounded each quarter and there are 4 quarters in a year, $n = 4$.

Plug the values into the formula.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = 2{,}000\left(1 + \frac{0.08}{4}\right)^{4\times3} =$$
$$2{,}000(1 + 0.02)^{12} = 2{,}000(1.02)^{12}$$

9. A

Determine the components of exponential growth:

Model M

Time interval is per month for 6 months. Hence, $m = 6$.

$$M = 400(2)^m = 400(2)^6 = 25{,}600$$

Model Q

Time interval is per quarter. Since there are 2 quarters in 6 months, $q = 2$.

$$Q = 2{,}100(3)^q = 2{,}100(3)^2 = 18{,}900$$

Determine difference: $25{,}600 - 18{,}900 = 6{,}700$.

10. C

Determine the components of exponential growth:

$a = a$.

$r = 2$.

Since the increase is every 29 years, time interval $= \frac{t}{29}$.

$y = n =$ trees after t years.

$$y = a(b)^x \rightarrow n = a(2)^{\frac{t}{29}}$$

11. D

Determine the components of compound interest formula:

Jenny

$P = d$. $r = 6\% = 0.06$. $t = 4$ years. $A = X$.

Since interest rate is compounded semi-annually, $n = 2$.

$$A = P\left(1 + \frac{r}{n}\right)^{nt} = d\left(1 + \frac{0.06}{2}\right)^{2\times 4} =$$
$$d(1 + 0.03)^8 = d(1.03)^8$$

Sara

$P = d$. $r = 5\% = 0.05$. $t = 4$ years. $A = Y$.

Since interest rate is compounded annually, $n = 1$.

$A = P(1 + r)^t = d(1 + 0.05)^4 = d(1.05)^4$.

Determine $X - Y$:

$$d(1.03)^8 - d(1.05)^4$$

12. C

Determine the components of compound interest formula:

$P = d$. $r = 10\% = 0.1$. $t = 3$ years.

Amount at the end of 3 years $= A = \$1{,}331$.

Since interest rate is compounded annually, $n = 1$.

$$A = P(1 + r)^t \rightarrow 1{,}331 = d(1.1)^3 \rightarrow$$
$$1{,}331 = d(1.331) \rightarrow d = \frac{1{,}331}{1.331} = \$1{,}000$$

Category 51 – Graphs of Exponential Growth and Decay Functions

1. B

Determine the graph that passes through the points $(0, a)$ and $(1, ab)$

$a = 1$. $b = 3$.

$(0, a) = (0, 1)$ and $(1, ab) = (1, 3)$.

Only, the graph in answer choice B passes through the above points.

2. C

Determine the function $g(x)$:

3 units right of $f(x)$ is

$$g(x) = 6(2)^{x-3} + \frac{11}{3}$$

Simplify.

$$g(x) = 6(2)^x(2)^{-3} + \frac{11}{3} \rightarrow$$
$$g(x) = 6(2)^x\left(\frac{1}{8}\right) + \frac{11}{3} \rightarrow$$
$$g(x) = \frac{6}{8}(2)^x + \frac{11}{3} \rightarrow$$
$$g(x) = \frac{3}{4}(2)^x + \frac{11}{3}$$

3. A

Identify the rate of change:

When m is increased by 1, n is increased 4-fold.

Hence, in the exponential form $f(x) = a(b)^x$, the rate of change $= b = 4$. This matches answer choice A.

4. 1.5 or 3/2

Determine the y-intercept of $f(x)$:

$+11$ is the shift of the y-coordinate up 11 units.

Since the y-coordinate of $f(x) + 11$ is 49.5, the y-coordinate of $f(x)$ is $49.5 - 11 = 38.5$.

Hence, y-coordinate $= a = 38.5$.

Determine the value of b: $a + b = 40$.

$$a + b = 40 \rightarrow 38.5 + b = 40 \rightarrow b = 1.5 = \frac{3}{2}$$

Section 10 – Drill

1. <u>A</u>

Determine the components of exponential growth:

$b = 1.03 = 1 + r = 1 + 0.03$.

Hence, $r = 0.03 = 3\% = k$.

2. <u>A</u>

Since the population decreased exponentially each year, it is exponential decay. The graph will decrease sharply from left to right as a curve. This matches with answer choice A.

3. <u>C</u>

Factor: $x^2 - 1$ can be factored as $(x-1)(x+1)$.

$$\sqrt[4]{(x-1)(x+1)}$$

Substitute $(x-1) = 16$ and replace root with exponential fraction:

$$\sqrt[4]{16(x+1)} \rightarrow \sqrt[4]{2^4(x+1)} \rightarrow$$

$$2^{4\times\frac{1}{4}}(x+1)^{\frac{1}{4}} \rightarrow 2(x+1)^{\frac{1}{4}}$$

4. <u>B</u>

4% as a decimal is 0.04. The value of $f(a)$ will increase with the value of a. The increase is by 4% for any value of a. Hence, it is linear increase.

5. <u>D</u>

Determine the components of exponential decay:

$y =$ end number.

Since b is less than 1, the rate of change is exponential decay. Hence, y will decrease with increase in time t.

6. <u>1</u>

Replace roots with fractional exponents:

$$\frac{(x^7)^{\frac{1}{3}}}{(x^4)^{\frac{1}{3}}} = x^{mn} \rightarrow \frac{x^{\frac{7}{3}}}{x^{\frac{4}{3}}} = x^{mn}$$

Move the expression in the denominator to the numerator:

$$x^{\frac{7}{3}} \times x^{-\frac{4}{3}} = x^{mn} \rightarrow x^{\frac{7-4}{3}} = x^{mn} \rightarrow x^{\frac{3}{3}} = x^{mn}$$

Equate the exponents and simplify:

$$mn = 1$$

7. <u>2</u>

Determine the vertical translation of the graph:

In the equation, $a = 4$.

Minus c is a downward translation of a. In the given graph, the y-coordinate of the y-intercept $= 2$. This is a downward translation of $a = 4$ by 2 units. Hence, $c = 2$.

8. <u>B</u>

Determine the components of exponential decay:

$a = n$.

$r = 3\%$. $b = 1 - r = 1 - 0.03 = 0.97$.

Since the decrease is every 8 days, time interval $= \frac{d}{8}$.

$P =$ population after d days.

$$y = a(b)^x \rightarrow P = n(0.97)^{\frac{d}{8}}$$

9. <u>6.75 or 27/4</u>

Replace root with fractional exponent and simplify:

$$(x^6)\left(x^{\frac{3}{4}}\right) = x^a \rightarrow x^{6+\frac{3}{4}} = x^a$$

Equate exponents:

$$a = 6 + \frac{3}{4} = \frac{27}{4} = 6.75$$

10. <u>B</u>

Determine the components of exponential decay:

$r = 0.5 = 50\% =$ half.

Time interval $= \frac{m}{20}$ is once every 20 minutes.

11. <u>C</u>

Evaluate each answer choice: The answer choices are given in the form of an exponential function, $g(x) = a(b)^x$, where a is the coefficient, and b is the base. Plug $x = 4$ in each answer choice and determine which value results in $g(x) = a$ or b.

Answer choice A: $g(x) = 4(1.6)^x = 4(1.6)^4$. Hence, $g(x) \neq 4$ or 1.6.

Answer choice B: $g(x) = 4(1.6)^{4-3} = 4(1.6)^1 = 4(1.6)$. Hence, $g(x) \neq 4$ or 1.6.

Answer choice C: $g(x) = 116(1.6)^{4-4} = 116(1.6)^0 = 116(1) = 116$. Hence, $g(x) = 116 = a$.

12. <u>D</u>

Determine the components of compound interest formula:

$P = d$.

$r = 10\% = 0.1$. $1 + r = 1 + 0.1 = 1.1$.

$t = 3$ years. $n = 1$.

$A =$ amount after 3 years $= d + 1{,}665$.

$$A = P(1+r)^t \rightarrow d + 1{,}655 = d(1.1)^3$$

Solve for d:

$$d + 1{,}655 = d(1.331) \rightarrow 1.331d - d = 1{,}655 \rightarrow$$

$$0.331d = 1{,}655 \rightarrow d = \$5{,}000$$

Section 11 – Manipulate Expressions and Equations

Category 52 – Fractions with Expressions in the Denominator

1. <u>C</u>

Determine the strategy and solve: Since the denominators have same expression, the numerators can be added.

$$\frac{3(x+1)-(3x+4)}{x-2}=1 \rightarrow$$

$$\frac{3x+3-3x-4}{x-2}=1 \rightarrow \frac{-1}{x-2}=1 \rightarrow$$

$$-1=x-2 \rightarrow x=-1+2 \rightarrow x=1$$

Check for an extraneous solution: 1 is not an extraneous solution.

2. <u>3</u>

Determine the strategy and solve: Since the fractions have the same denominator, the numerators can be added.

$$\frac{3a^2+7ab}{(a+b)}-\frac{ab-3b^2}{(a+b)}=9 \rightarrow$$

$$\frac{3a^2+7ab-ab+3b^2}{(a+b)}=9 \rightarrow$$

$$\frac{3a^2+6ab+3b^2}{(a+b)}=9 \rightarrow \frac{a^2+2ab+b^2}{(a+b)}=3$$

$(a^2+2ab+b^2)$ can be factored as $(a+b)(a+b)$.

$$\frac{(a+b)(a+b)}{(a+b)}=3 \rightarrow \frac{(a+b)\cancel{(a+b)}}{\cancel{(a+b)}}=3 \rightarrow$$

$$a+b=3$$

3. <u>8</u>

Determine the strategy and solve: y^2-9 can be factored as $(y+3)$ and $(y-3)$.

$$\frac{(y-3)(y+3)}{(y+3)}=5 \rightarrow \frac{(y-3)\cancel{(y+3)}}{\cancel{(y+3)}}=5 \rightarrow$$

$$y-3=5 \rightarrow y=5+3=8$$

Check for an extraneous solution: 8 is not an extraneous solution.

4. <u>9</u>

Determine the strategy and solve: Multiplying the numerator and denominator of the left fraction with $(x+y)$ will result in (x^2-y^2). The given values of $(x+y)$ and (x^2-y^2) can be substituted to solve.

$$\frac{2(x+y)}{(x+y)(x-y)}+\frac{5}{x^2-y^2} \rightarrow$$

$$\frac{2(x+y)}{x^2-y^2}+\frac{5}{x^2-y^2}$$

Substitute given values of $(x+y)$ and (x^2-y^2).

$$\frac{2\times 11}{3}+\frac{5}{3}=\frac{22}{3}+\frac{5}{3}=\frac{27}{3}=9$$

5. <u>D</u>

Determine the strategy and solve: Multiplying the numerator and denominator of the left fraction with $(x+1)$ will create the same denominator, x^2-1.

$$\frac{3(x+1)}{(x+1)(x-1)}+\frac{k}{x^2-1}=\frac{3x+8}{x^2-1} \rightarrow$$

$$\frac{3(x+1)}{x^2-1}+\frac{k}{x^2-1}=\frac{3x+8}{x^2-1} \rightarrow$$

$$3(x+1)+k=3x+8 \rightarrow 3x+3+k=3x+8 \rightarrow$$

$$k=3x+8-3x-3 \rightarrow k=5$$

6. <u>B</u>

Determine the strategy and solve: Factor common terms in the denominator and create one equation.

$$\frac{m-4}{n(n-4)}+\frac{\cancel{m}(n-5)}{\cancel{m}n(n-4)} \rightarrow \frac{m-4}{n(n-4)}+\frac{(n-5)}{n(n-4)}$$

Since the denominators are the same, the two fractions can be added.

$$\frac{m-4+n-5}{n(n-4)} \rightarrow \frac{m+n-9}{n^2-4n}$$

7. <u>A</u>

Determine the strategy and solve: The two denominators on the left-side of the equation, $(x+5)$ and $(x-2)$, are multiples of the denominator on the right-side equation $(x+5)(x-2)$. Multiplying the numerator and denominator of the left-side equation with the missing common multiple will make all the denominators the same.

$$\frac{6(x-2)}{(x-2)(x+5)}+\frac{3(x+5)}{(x+5)(x-2)}=\frac{7x+9}{(x+5)(x-2)} \rightarrow$$

$$6(x-2)+3(x+5)=7x+9 \rightarrow$$

$$6x-12+3x+15=7x+9 \rightarrow$$

$$6x+3x-7x=9-15+12 \rightarrow 2x=6 \rightarrow x=3$$

Check for an extraneous solution: 3 is not an extraneous solution.

Category 53 – Rearrange Variables in an Equation

1. <u>C</u>

Rearrange to isolate l:

Subtract m from both sides.

$$5k - m = \cancel{m - m} + 14l \;\rightarrow\; 5k - m = 14l$$

Divide both sides by 14.

$$\frac{5k - m}{14} = \frac{\cancel{14}\,l}{\cancel{14}} \rightarrow \frac{5k - m}{14} = l$$

2. <u>A</u>

Rearrange to isolate π:

Divide both sides by $(1 + r)$ to isolate $(1 + \pi)$.

$$\frac{(1+i)}{(1+r)} = \frac{\cancel{(1+r)}(1+\pi)}{\cancel{(1+r)}} \rightarrow$$

$$(1 + \pi) = \frac{(1+i)}{(1+r)}$$

Subtract 1 from both sides to isolate π.

$$\cancel{1} - \cancel{1} + \pi = \frac{(1+i)}{(1+r)} - 1 \rightarrow$$

$$\pi = \frac{(1+i)}{(1+r)} - 1$$

3. <u>C</u>

Rearrange to isolate s (since all the answer choices are in terms of s):

Move fractions to one side of the equation.

$$\left(\frac{ut + 2u + 1}{u + 1}\right) - \left(\frac{us - ut^2}{u + 1}\right) = 1$$

Write as one fraction as both denominators are the same.

$$\frac{ut + 2u + 1 - us + ut^2}{u + 1} = 1$$

Cross multiply.

$$ut + 2u + 1 - us + ut^2 = u + 1$$

Move all terms without s to one side of the equation and simplify.

$$us = ut + 2u + 1 + ut^2 - u - 1 \rightarrow$$

$$us = ut + u + ut^2$$

Factor u in the right-side expression.

$$us = u(t + 1 + t^2)$$

Divide both sides by u.

$$s = t + 1 + t^2 = t^2 + t + 1$$

4. <u>A</u>

Rearrange to isolate μ:

Multiply both sides by $\pi \times r^4$ to remove them from the denominator of the fraction.

$$(\pi \times r^4) \times R = \frac{(8 \times \mu)l}{\cancel{\pi \times r^4}} \times \cancel{(\pi \times r^4)} \rightarrow$$

$$\pi \times r^4 \times R = 8 \times \mu \times l$$

Divide both sides by $8 \times l$ to isolate μ.

$$\frac{\pi \times r^4 \times R}{8 \times l} = \frac{\cancel{8 \times l} \times \mu}{\cancel{8 \times l}} \rightarrow$$

$$\frac{\pi r^4 R}{8l} = \mu$$

5. <u>B</u>

Rearrange to isolate r:

Multiply both sides by r^2 and divide by F to isolate r^2.

$$\frac{r^2}{\cancel{F}} \times \cancel{F} = G\frac{m_1 m_2}{\cancel{r^2}} \times \frac{\cancel{r^2}}{F} \rightarrow r^2 = G\frac{m_1 m_2}{F}$$

Remove the square from r.

$$r = \sqrt{G\frac{m_1 m_2}{F}}$$

Category 54 – Combine and Factor Like Terms

1. <u>B</u>

Simplify and add/subtract like terms:

$$2(x^2 - 1) + 10x^3 - 2x^2 \rightarrow 2x^2 - 2 + 10x^3 - 2x^2$$

$$\rightarrow 10x^3 - 2$$

2. <u>D</u>

Simplify and add/subtract like terms:

$$m^2 + 3n^3 - 2mn - m(m - 2) \rightarrow$$

$$m^2 + 3n^3 - 2mn - m^2 + 2m \rightarrow$$

$$3n^3 - 2mn + 2m$$

3. <u>A</u>

Simplify and add/subtract like terms:

$$4x(x^3 + 3x^2) - 3(x^4 - 2x^3) + x^4 \rightarrow$$

$$4x^4 + 12x^3 - 3x^4 + 6x^3 + x^4 \rightarrow 2x^4 + 18x^3$$

Factor like terms: $2x^3$ can be factored.

$$2x^3(x + 9)$$

4. <u>C</u>

Add/subtract like terms:

$$s^2t - 3t^3 + 3s^2t + 5t^3 + t \rightarrow 4s^2t + 2t^3 + t$$

Factor like terms: t can be factored.

$$t(4s^2 + 2t^2 + 1)$$

Category 55 – Expressions with Square Root

1. D

Square both sides: Move 2 to the right-side before squaring.

$\sqrt{x+5}-2 = 3 \ \rightarrow \ \sqrt{x+5} = 3+2 \ \rightarrow \ \sqrt{x+5} = 5$

Now square both sides.

$$\left(\sqrt{x+5}\right)^2 = 5^2 \ \rightarrow \ x+5 = 25$$

Solve:

$$x = 25-5 \ \rightarrow \ x = 20$$

2. B

Square both sides: Move 1 to the right-side before squaring.

$$\sqrt{\frac{27}{x}}+1 = 4 \ \rightarrow \ \sqrt{\frac{27}{x}} = 4-1 \ \rightarrow \ \sqrt{\frac{27}{x}} = 3$$

Now square both sides.

$$\left(\sqrt{\frac{27}{x}}\right)^2 = 3^2 \ \rightarrow \ \frac{27}{x} = 9$$

Solve:

$$9x = 27 \ \rightarrow \ x = 3$$

3. C

Square both sides:

$$\left(\sqrt{x^2+5}\right)^2 = 3^2 \ \rightarrow \ x^2+5 = 9$$

Solve:

$$x^2 = 9-5 \rightarrow x^2 = 4 \ \rightarrow \ x = \pm 2$$

Since $x > 0$, $x = 2$.

4. 2

Square both sides:

$$\left(\sqrt{5x-1}\right)^2 = \left(3\sqrt{x-1}\right)^2 \ \rightarrow \ 5x-1 = 9\,(x-1)$$

Solve:

$$5x-1 = 9x-9 \ \rightarrow \ 9x-5x = 9-1 \ \rightarrow$$
$$4x = 8 \ \rightarrow \ x = 2$$

5. 1

Square both sides: Move $\sqrt{x+26}$ to the right-side for ease.

$$\left(3\sqrt{x+2}\right)^2 = \left(\sqrt{x+26}\right)^2 \ \rightarrow \ 9(x+2) = (x+26)$$

Solve:

$$9x+18 = x+26 \ \rightarrow \ 9x-x = 26-18 \ \rightarrow$$
$$8x = 8 \ \rightarrow \ x = 1$$

6. B

Square both sides: Move 2 to the right-side before squaring.

$$\sqrt{7-2x}+2 = x \ \rightarrow \ \sqrt{7-2x} = x-2$$

Now square both sides.

$$\left(\sqrt{7-2x}\right)^2 = (x-2)^2 \ \rightarrow \ 7-2x = (x-2)^2$$

Solve: FOIL the right-side expression.

$$7-2x = x^2-4x+4$$

Form a quadratic equation.

$$x^2-4x+4+2x-7 = 0 \ \rightarrow \ x^2-2x-3 = 0$$

Factor.

$$(x+1)(x-3) = 0$$

Hence $x = -1$ and 3.

Check for an extraneous solution:

Plug $x = -1$ into the equation.

$$\sqrt{7-2x}+2 = x \ \rightarrow \ \sqrt{7-2(-1)}+2 = -1 \ \rightarrow$$
$$\sqrt{9}+2 = -1 \ \rightarrow \ 3+2 = -1$$

$x = -1$ is an extraneous solution since both sides of the equation are not equal.

Plug $x = 3$ into the equation.

$$\sqrt{7-2x}+2 = 3 \ \rightarrow \ \sqrt{7-2(3)}+2 = 3 \ \rightarrow$$
$$\sqrt{7-6}+2 = 3 \ \rightarrow \ 1+2 = 3 \ \rightarrow \ 3 = 3$$

$x = 3$ is not an extraneous solution.

7. 2

Square both sides:

$$\left(\sqrt{(x-1)^2}\right)^2 = \left(\sqrt{(15-7x)}\right)^2 \ \rightarrow$$
$$(x-1)^2 = 15-7x$$

FOIL and create a quadratic equation:

$$x^2-2x+1 = 15-7x \ \rightarrow$$
$$x^2-2x+1+7x-15 = 0 \ \rightarrow$$
$$x^2+5x-14 = 0$$

Factor.

$$(x-2)(x+7) = 0$$

Hence, the two values of x are 2 and -7.

The largest solution is 2.

Check for an extraneous solution: Largest solution $x = 2$ is not an extraneous solution.

8. B

Solve for x: Substitute the given value of $f(x)$ into the equation.

$$\sqrt{96} = 4\sqrt{3x} \ \rightarrow \ \sqrt{16 \times 6} = 4\sqrt{3x} \ \rightarrow$$
$$4\sqrt{6} = 4\sqrt{3x} \ \rightarrow \ \sqrt{6} = \sqrt{3x} \ \rightarrow \ (\sqrt{6})^2 = (\sqrt{3x}\,)^2 \ \rightarrow$$
$$6 = 3x \ \rightarrow \ 2 = x$$

Section 11 – Drill

1. <u>D</u>

Simplify and add/subtract like terms:

$$4a(a^2+b)-2a^2-ab \rightarrow$$
$$4a^3+4ab-2a^2-ab \rightarrow 4a^3-2a^2+3ab$$

2. <u>D</u>

Square both sides:

$$\left(\sqrt{2k+17}\right)^2 = 7^2 \rightarrow 2k+17 = 49$$

Solve:

$$2k = 49-17 \rightarrow 2k = 32 \rightarrow k = 16$$

When using Desmos graphing calculator, type x for k.

3. <u>B</u>

Simplify and add/subtract like terms:

$$x(2x^2+y)-2x(x^2-y^2) \rightarrow$$
$$2x^3+xy-2x^3+2xy^2 \rightarrow xy+2xy^2 \rightarrow$$
$$xy(1+2y)$$

4. <u>5</u>

Determine the strategy and solve: Since the denominators have same expression, the numerators can be added.

$$\frac{2y^2+y-y^2-5y+4}{y-2} = 3 \rightarrow \frac{y^2-4y+4}{y-2} = 3$$

(y^2-4y+4) can be factored as $(y-2)(y-2)$.

$$\frac{(y-2)(y-2)}{y-2} = 3 \rightarrow \frac{(y-2)\cancel{(y-2)}}{\cancel{y-2}} = 3 \rightarrow$$
$$y-2 = 3 \rightarrow y = 5$$

Check for an extraneous solution: 5 is not an extraneous solution.

5. <u>B</u>

Rearrange to isolate q:

Add q to both sides to remove negative value.

$$91p\cancel{-q+q} = 125r+q \rightarrow 91p = 125r+q$$

Subtract $125r$ from both sides.

$$91p-125r = \cancel{125r-125r}+q \rightarrow$$
$$q = 91p-125r$$

6. <u>B</u>

Rearrange to isolate a:

Multiply all terms on both sides by 2 to remove $\frac{1}{2}$ from the right-side expression.

$$2\times m = \cancel{2}\times\left(\frac{1}{\cancel{2}}am\right)+(2\times 4) \rightarrow 2m = am+8$$

Subtract 8 from both sides to isolate am.

$$2m-8 = am+\cancel{8}-\cancel{8} \rightarrow 2m-8 = am$$

Divide both sides by m to isolate a.

$$\frac{2m-8}{m} = \frac{a\cancel{m}}{\cancel{m}} \rightarrow \frac{2m-8}{m} = a \rightarrow$$
$$\frac{2m}{m}-\frac{8}{m} = a \rightarrow a = 2-\frac{8}{m}$$

7. <u>D</u>

Rearrange to isolate H:

Multiply all terms on both sides by 2 to remove $\frac{1}{2}$ from the right-side expression.

$$2\times K = \cancel{2}\times\frac{1}{\cancel{2}}(D^2+H)+(2\times D^2) \rightarrow$$
$$2K = D^2+H+2D^2 \rightarrow 2K = H+3D^2$$

Subtract $3D^2$ from both sides to isolate H.

$$2K-3D^2 = H\cancel{+3D^2-3D^2} \rightarrow H = 2K-3D^2$$

8. <u>5</u>

Determine the strategy and solve: x^2-9 can be factored as $(x+3)$ and $(x-3)$.

$$\frac{(x-3)(x+3)}{2(x-3)} = 4 \rightarrow \frac{\cancel{(x-3)}(x+3)}{2\cancel{(x-3)}} = 4 \rightarrow$$
$$\frac{x+3}{2} = 4 \rightarrow x+3 = 4\times 2 \rightarrow x = 8-3 = 5$$

Check for an extraneous solution: 5 is not an extraneous solution.

9. <u>B</u>

Move 1 to the right-side of the equation.

Note that the entire left-side expression can be squared.

$$\sqrt{\frac{36}{4x^2}}-1 = 0 \rightarrow \sqrt{\frac{36}{4x^2}} = 1 \rightarrow \sqrt{\left(\frac{6}{2x}\right)^2} = 1$$

The square root can be removed. 1 and 1^2 are same.

Solve:

$$\frac{6}{2x} = 1 \rightarrow 2x = 6 \rightarrow x = 3$$

Check for an extraneous solution: 3 is not an extraneous solution.

10. B

Square both sides:

$$\left(\sqrt{5c^2-4}\right)^2 = (2c)^2 \ \rightarrow\ 5c^2-4 = 4c^2$$

Solve:

$$5c^2-4c^2 = 4 \ \rightarrow\ c^2 = 4 \ \rightarrow\ c = \pm 2$$

Hence, the two values of c are -2 and 2.

Check for an extraneous solution:

Plug $c=-2$ into the given equation.

$$\sqrt{5(-2)^2-4} = (2\times -2) \ \rightarrow\ \sqrt{20-4} = -4 \ \rightarrow$$
$$\sqrt{16} = -4 \ \rightarrow\ 4 = -4$$

$c=-2$ is an extraneous solution since both sides of the equation are not equal.

Plug $c=2$ into the given equation.

$$\sqrt{5(2)^2-4} = (2\times 2) \ \rightarrow\ \sqrt{20-4} = 4 \ \rightarrow$$
$$\sqrt{16} = 4 \ \rightarrow\ 4 = 4$$

$c=2$ is not an extraneous solution.

When using Desmos graphing calculator, type x for c.

11. A

Determine the strategy and solve: a^2-b^2 can be factored as $(a-b)$ and $(\ a+b)$. The given values of $(a-b)$ and $(a+b)$ can be substituted to solve the equation.

$$\frac{1}{(a-b)(a-b)(a+b)} \rightarrow \frac{1}{(2)(2)(3)} \rightarrow \frac{1}{12}$$

12. 4

Determine the strategy and solve: Simplify the fractions in the denominator first and then solve the equation.

Simplify the denominator.

Create the same denominator using the common multiples $(x-2)$ and $(x+2)$.

$$\frac{1}{x+2}+\frac{1}{x-2} \ \rightarrow\ \frac{(x-2)}{(x+2)(x-2)}+\frac{(x+2)}{(x+2)(x-2)} \ \rightarrow$$
$$\frac{(x-2)+(x+2)}{(x+2)(x-2)} \ \rightarrow\ \frac{2x}{(x^2-4)}$$

Substitute the simplified denominator in the given equation and solve.

$$\frac{2}{\frac{2x}{(x^2-4)}} = x-1 \ \rightarrow\ 2\times\frac{(x^2-4)}{2x} = x-1 \ \rightarrow$$
$$\frac{(x^2-4)}{x} = x-1 \ \rightarrow\ x^2-4 = x(x-1) \ \rightarrow$$
$$x^2-4 = x^2-x \ \rightarrow\ -4 = -x \ \rightarrow\ x = 4$$

Check for an extraneous solution: 4 is not an extraneous solution.

13. A

Rearrange to isolate v_o:

Subtract $\frac{at^2}{2}$ and s_o from both sides to isolate $v_o t$.

$$s-\left(\frac{at^2}{2}\right)-s_o = \cancel{\frac{at^2}{2}}-\left(\cancel{\frac{at^2}{2}}\right)+v_o t+\cancel{s_o}-\cancel{s_o} \ \rightarrow$$
$$s-\frac{at^2}{2}-s_o = v_o t$$

Divide both sides by t to isolate v_o.

$$\frac{s}{t}-\frac{at\cancel{t}}{2\cancel{t}}-\frac{s_o}{t} = \frac{v_o\cancel{t}}{\cancel{t}} \ \rightarrow\ \frac{s}{t}-\frac{at}{2}-\frac{s_o}{t} = v_o$$

Add fractions with t in the denominator.

$$v_o = \frac{s-s_o}{t}-\frac{at}{2}$$

14. 2

Determine the strategy and solve: a^2-b^2 can be factored as $(a+b)(a-b)$ and $2a^2+4ab+2b^2$ can be factored as $2(a+b)(a+b)$.

$$(a+b)(a-b) = \frac{2(a+b)(a+b)}{a+b} \ \rightarrow$$
$$(a+b)(a-b) = \frac{2(a+b)\cancel{(a+b)}}{\cancel{a+b}} \ \rightarrow$$
$$\cancel{(a+b)}(a-b) = 2\cancel{(a+b)} \ \rightarrow$$
$$a-b = 2$$

15. −8

Square both sides:

$$\left(\sqrt{(x+2)^2}\right)^2 = \left(\sqrt{(28-x)}\right)^2 \ \rightarrow\ (x+2)^2 = 28-x$$

FOIL and create a quadratic equation:

$$x^2+4x+4 = 28-x \ \rightarrow\ x^2+4x+4+x-28 = 0$$
$$x^2+5x-24 = 0$$

Factor.

$$(x-3)(x+8)$$

Hence, the two values of x are 3 and -8.

The smallest solution is -8.

Check for an extraneous solution: $x=-8$.

Plug $x=-8$ into the given equation.

$$\sqrt{(-8+2)^2} = \sqrt{28-(-8)} \ \rightarrow\ \sqrt{(-6)^2} = \sqrt{36} \ \rightarrow$$
$$\sqrt{36} = \sqrt{36} \ \rightarrow\ 6 = 6$$

$c=-8$ is not an extraneous solution since both sides of the equation are equal.

Section 12 – Data Analysis and Interpretation

Category 56 – Probability

1. <u>B</u>

Determine the probability:

$$\frac{\text{all faces of 5}}{\text{all faces}} \rightarrow \frac{2}{6} = \frac{1}{3}$$

2. <u>C</u>

Determine the probability:

$$\frac{\text{all red ribbons}}{\text{all ribbons}} = \frac{n}{n+25}$$

3. <u>C</u>

Determine the probability:

$$\frac{\text{all blue electric cars}}{\text{all blue cars}} = \frac{311}{516}$$

4. <u>46</u>

Determine the number of mystery novels:

$$\frac{4}{7} \times 140 = 80$$

Determine the number of adventure novels: Set up the probability.

$$\frac{\text{all adventure novels}}{140} = \frac{1}{10} \rightarrow$$

$$\text{all adevnture novels} = \frac{140}{10} = 14$$

Determine the number of science fiction novels:

$$140 = 80 + 14 + \text{science fiction novels} \rightarrow$$

$$\text{science fiction novels} = 140 - 94 = 46$$

5. <u>16/80 or 8/40 or 4/20 or 1/5 or 0.2</u>

Determine the probability:

$$\frac{\text{all small blue marbles} + \text{all medium red marbles}}{\text{all marbles} - \text{all large marbles}} =$$

$$\frac{8+8}{100-20} = \frac{16}{80} = \frac{1}{5} = 0.2$$

6. <u>3</u>

Determine the probability:

$$\frac{\text{all Team A participants for Coffee Brand B}}{\text{all participants}} = \frac{1}{9}$$

Solve for a:

$$\frac{a}{6+a+2+5+5+6} = \frac{1}{9} \rightarrow \frac{a}{24+a} = \frac{1}{9} \rightarrow$$

$$9a = 24 + a \rightarrow 8a = 24 \rightarrow a = 3$$

Category 57 – Graphs with Line Segments and Curves

1. <u>D</u>

Follow the events from the graph:

Quinn drank the protein shake over 90 mins. This eliminates answer choices B and C that show the entire protein shake finished in 60 and 30 minutes, respectively.

Quinn finished half the protein shake within 30 minutes and then waited for 30 minutes. The wait is represented by a horizontal line (time). This eliminates answer choice A that shows 60 minutes of wait.

Answer choice D matches the events.

2. <u>C</u>

Read the data points from the graph: From the graph, it can be observed that week 8 and 9 will give the lowest total.

If unsure, evaluate each answer choice. See below.

Answer choice A: week $1 +$ week $2 = 45 + 10 = 55$.

Answer choice B: week $7 +$ week $8 = 25 + 15 = 40$.

Answer choice C: week $8 +$ week $9 = 15 + 15 = 30$.

Answer choice D: week $18 +$ week $19 = 40 + 7 = 47$.

3. <u>A</u>

Read the data points from the graph: The graph shows that the greatest difference is from 2 pm to 3 pm.

If unsure, read the data point for each hour and calculate the difference.

4. C

Read the data points from the graph: Since the speed is along the vertical axis, a horizontal line indicates no change in speed. The horizontal lines are from 20 to 35 miles and from 50 to 70 miles.

Hence, the total distance in miles Geeta traveled at a constant speed is

$$15 + 20 = 35$$

5. B

Read the data points from the graph: No need to read all the data points. For each month, check if one data point is double the other.

In August, the rainfall in 2018 (8 inches) was twice that of 2017 (4 inches). (Note that in May the rainfall in 2017 was twice that of 2018. This is the opposite.)

6. B

Read the population from the graph: The graph shows that the greatest increase was from 1984 to 1988.

If unsure, read the population for each given four-year period and determine the difference.

7. C

Read the initial and ending weight from the graph: The graph shows that Tony lost greater weight than Sam. See calculation below.

Tony: initial weight = 174, ending weight = 155.
Difference = $174 - 155 = 19$.

Sam: initial weight = 165, ending weight = 155.
Difference = $165 - 155 = 10$.

Category 58 – Scatter Plots and Lines of Best Fit

1. C

Read the data point on the line of best fit:

2012 (horizontal axis) corresponds to 120 grams of wheat production (vertical axis).

2. 4

Read the data point above the line of best fit:

Total points = 4.

3. D

The time is represented along the horizontal axis, and profit is represented along the vertical axis.

At y-intercept, time = 0. Hence, it is the predicted monthly profit in thousands of dollars when no time is spent on advertising.

4. A

Determine the slope of the line of best fit:

Below is slope using the points (60, 80) and (100, 60).

$$\frac{60 - 80}{100 - 60} = \frac{-20}{40} = -0.5$$

This eliminates answer choices C and D. From the graph, it can be observed that that the y-intercept is between 100 and 120. This eliminates answer choice B.

5. B

The relationship between age and body mass is the slope of the line of best fit. Age is represented along the horizontal axis (run) and body mass is represented along the vertical axis (rise).

Determine the slope of the line of best fit:

Below is slope using the points (20, 10) and (40, 15).

$$\frac{15 - 10}{40 - 20} = \frac{5}{20} = \frac{\text{rise}}{\text{run}} = \frac{5 \text{ kilograms body mass}}{20 \text{ days in age}}$$

Hence, for every 20 days in age, the estimated (predicted) body mass increase is 5 kilograms.

6. A

Determine the slope of the line of best fit:

The relationship between weight loss and number of weeks in the program is the slope of the line of best fit.

Below is slope using the points (8, 3) and (14, 5).

$$\frac{5 - 3}{14 - 8} = \frac{2}{6} = \frac{\text{rise}}{\text{run}} = \frac{2 \text{ pounds weight loss}}{6 \text{ weeks in program}} = \frac{0.33}{1}$$

Hence, for every 1 week, the predicted weight loss is 0.33 pounds.

Category 59 – Bar Graphs

1. A

The graph shows that the greatest change is between week 1 and 2.

2. B

The graph shows that Dixie read twice the number of mystery books as Tracy. If unsure, read the height of each bar and compare.

3. A

Read the height of the top bar for May: Since there are 100 students, the percent can be directly read from the vertical axis.

25 students responded "No" $= 25\%$.

4. 80

Read the height of each top bar:

$1 = 10.$ $2 = 20.$ $3 = 20.$ $4 = 10.$ $5 = 15.$ $6 = 5.$

$$10 + 20 + 20 + 10 + 15 + 5 = 80$$

Category 60 – Histograms and Dot Plots

1. A

Count the dots on 3 and 4 times a week and add them:

$$5 + 4 = 9$$

2. B

Read the heights of bars 3, 4, 5, and 6 and add them:

$$15 + 10 + 5 + 5 = 35$$

Determine the percent:

$$\frac{35}{70} \times 100 = 50\%$$

3. 4

Count the number of dots on each group: The largest number of students are enrolled in 4 clubs.

4. C

Read the heights of bars 20-30 and 30-40 and add them:

Bar 20-30 $= 20.$ Bar 30-40 $= 10.$

$$20 + 10 = 30$$

Category 61 – Mean

1. 46.75

Set up the mean:

Count of numbers $= 8$.

$$\frac{20 + 35 + 42 + 50 + 54 + 56 + 57 + 60}{8} = \frac{374}{8} = 46.75$$

2. B

Determine the sum of scores for both the classes:

Total number of students $= 40 + 20 = 60$.

The total score of 40 students in Mr. Daniel's class is

$$40 \times a = 40a$$

The total score of 20 students in Mr. Power's class is

$$20 \times b = 20b$$

The total score of both classes $= 40a + 20b$.

Determine the mean:

$$\frac{40a + 20b}{60} = \frac{20(2a + b)}{60} = \frac{1}{3}(2a + b)$$

3. 16

Set up the mean:

Mean of the 2 numbers $= 32$.

Let the lesser number $= x$. The bigger number $= 3x$.

$$\frac{x + 3x}{2} = 32 \rightarrow \frac{4x}{2} = 32 \rightarrow$$

$$2x = 32 \rightarrow x = 16$$

4. 104

Set up the mean:

Count of numbers after adding $y = 6$.

The mean of the 6 numbers $= 134$.

$$\frac{50 + 120 + 230 + 80 + 220 + y}{6} = 134 \rightarrow$$

$$\frac{700 + y}{6} = 134 \rightarrow 700 + y = 6 \times 134 \rightarrow$$

$$700 + y = 804 \rightarrow y = 804 - 700 = 104$$

5. 98

Set up the mean:

Count of numbers after adding 5^{th} test $= 5$.

Let the 5^{th} test $= x$.

Since the mean of the 5 tests must be at least 90, equate the mean to 90. This will give the minimum score Gina will need on the fifth biology test.

$$\frac{85 + 88 + 87 + 92 + x}{5} = 90 \rightarrow \frac{352 + y}{5} = 90$$

$$352 + x = 5 \times 90 \rightarrow 352 + x = 450 \rightarrow$$

$$x = 450 - 352 = 98$$

6. <u>10.5</u>

Determine the sum of weights:

Total number of dumbbells $= 6 + 10 + 4 = 20$.

Total weight of 20 dumbbells is

$$(6 \times 5) + (10 \times 10) + (4 \times 20) =$$
$$30 + 100 + 80 = 210$$

Determine the mean:

$$\frac{210}{20} = 10.5$$

7. <u>82</u>

Determine the sum of numbers:

Since the mean of 7 tests $= 92$, their sum $= 7 \times 92 = 644$.

Since the mean of first 5 tests $= 96$, their sum $= 5 \times 96 = 480$.

The sum of last 2 tests $= 644 - 480 = 164$.

Determine the mean of the last 2 tests:

$$\frac{164}{2} = 82$$

8. <u>43</u>

Determine the sum of numbers:

Since the mean of 12 numbers $= 38$, their sum $= 12 \times 38 = 456$.

After removing 11 and 15, the sum of the remaining 10 numbers $= 456 - 11 - 15 = 430$.

Determine the mean of the remaining 10 numbers:

$$\frac{430}{10} = 43$$

9. <u>144</u>

Determine the sum of the 12 integers in the data set:

$$(4 \times 210) + (4 \times 148) + (4 \times 74) =$$
$$840 + 592 + 296 = 1{,}728$$

Determine the mean of the 12 integers in the data set:

$$\frac{1{,}728}{12} = 144$$

Category 62 – Histograms, Dot Plots, and Mean

1. <u>C</u>

Determine the total number of family members in all the households:

$$(1 \times 4) + (2 \times 2) + (3 \times 14) + (4 \times 16) + (5 \times 8) +$$
$$(7 \times 2) + (8 \times 4) =$$
$$4 + 4 + 42 + 64 + 40 + 14 + 32 = 200$$

Determine the mean: There are 50 households.

$$\frac{200}{50} = 4$$

2. <u>B</u>

Determine the total number of kittens for all the cats:

$$(2 \times 1) + (3 \times 3) + (4 \times 6) + (5 \times 5) =$$
$$2 + 9 + 24 + 25 = 60$$

Determine the mean: There are 15 cats.

$$\frac{60}{15} = 4$$

3. <u>4</u>

Determine the total number of times all the buses were late:

$(1 \times 2) + (2 \times 4) + (3 \times 3) + (4 \times 4) + (5 \times 7) +$
$(6 \times 5) = 2 + 8 + 9 + 16 + 35 + 30 = 100$

Determine the mean: There are 25 buses.

$$\frac{100}{25} = 4$$

4. <u>3</u>

Determine the total number of vacation days for all the employees:

$(1 \times 9) + (2 \times 10) + (3 \times 12) + (4 \times 8) + (5 \times 7) +$
$(6 \times 2) =$

$$9 + 20 + 36 + 32 + 35 + 12 = 144$$

Determine the mean: There are 48 employees.

$$\frac{144}{48} = 3$$

Category 63 – Median

1. <u>25</u>

Sort the numbers least to greatest and determine the middle number:

Since there are 6 numbers in the set, the median is the average of the 3rd and 4th number.

$$\frac{24.3 + 25.7}{2} = \frac{50}{2} = 25$$

2. <u>A</u>

Sort the numbers least to greatest and determine the middle number:

The middle number $= 49.37$ inches. The corresponding year $= 2010$.

3. B

Determine the median: There are 27 employees. The salary range that includes the 14th employee is the median salary range.

There are 15 employees in the first salary range from \$40,000 - \$60,000. Hence, the 14th employee falls in this salary range. The median salary is any number between 40,000 and 60,000.

4. 35

Sort the numbers in both the sets from least to greatest and determine the middle number of each set: Since there are 7 numbers in the set, median is the 4th number.

For Set $1 = A$, the median is 13.5.

For Set $2 = B$, the median is 48.5.

Determine $B - A$: $48.5 - 13.5 = 35$

5. C

Sort the integers least to greatest: Since y is the median, it will be in the middle.

$$5, 8, y, 11, 15$$

Determine the possible values of y: Since y is between 8 and 11, the only possible integer values of y can be 8, 9, 10, or 11. Hence, option I and option II are correct.

6. A

Determine the median: Since there are 49 students, the group that includes the 25th student is the median group. Starting from the top of the table, continue a cumulative count of students till the 25th student is reached.

25th student corresponds to the group with 4 - 6 number of books. The median number of books could be any number from 4 to 6.

Category 64 – Histograms, Dot Plots, Bar Graphs, and Median

1. B

Determine the median: Since there are 100 households, the bar that includes the 50th and 51st household contains the median car expense.

This corresponds to the bar that is at least \$400 but less than \$500. Any number between this could be a median car expense.

2. C

Determine the median: Since there are 85 adults, the bar that includes the 43rd adult contains the median speed limit.

This corresponds to the bar that is at least 60 but less than 65. Any number between this could be a median speed.

3. B

Determine the median: Since there are 15 baby dolphins, the group that includes the 8th baby dolphin contains the median weight.

This corresponds to baby dolphin = 35 - 40 pounds. Any number between 35 and 40 could be a median weight.

4. 2

Determine the median: Since there are 60 employees, the bar that includes the 30th and 31st employee contains the median number of degrees.

This corresponds to bar $= 2$ degrees. Hence, the median number of degrees $= 2$.

5. 3

Determine the median: Since there are 21 quizzes, the bar that includes the 11th quiz contains the median score.

This corresponds to bar for quiz score $= 8$. Hence, the median quiz score $= 8$.

There are 3 quizzes with the median score $= 8$.

6. 9

Determine the median: Since there are 49 newborn babies, the bar that includes the 25th newborn baby contains the median length.

This corresponds to bar for length $= 21$ inches. Hence, the median length of the newborn babies $= 21$.

There are 9 newborn babies with the median length $= 21$.

Category 65 – Box Plots and Median

1. C

Read the median from the box plot:

The median is between 15 and 20 and closer to 20. Answer choice C is the closest choice.

2. A

Read the median from the box plot:

Median of School A $= 3.5$. Median of School B $= 3$.

Difference: $3.5 - 3 = 0.5$.

3. A

Determine the median: Since there are 192 data points, the median is between 96th and 97th response.

Rating 1: 1st to 9th response.

Rating 2: 10th to 59th response.

Rating 3: 60th to 86th response.

Rating 4: 87th to 127th response. Hence, median $= 4$.

This eliminates answer choices B and C. Remaining answer choices A and D have same $Q1$ but different $Q3$.

Determine $Q3$: The median of the 50% higher numbers fall in rating $= 5$. This eliminates answer choice D.

Category 66 – Standard Deviation and Range

1. <u>159</u>

Since the lowest number is reduced by 159, the difference (range) between the lowest and greatest number will change by 159.

2. <u>A</u>

Compare the standard deviation: From 2010 to 2016, the number of students enrolled in Lacrosse are between 0 and 55, and the number of students enrolled in Soccer are between 41 and 58. Hence, the students enrolled in Soccer have a lower spread and are closer to the mean.

3. <u>C</u>

Compare the standard deviation: The numbers in the set are between 6 and 31.

After adding 52, the numbers in the set will be between 6 and 52. This will increase the spread of the numbers.

4. <u>A</u>

Compare the range:

Set A: $21 - 15 = 6$.

Set B: $502 - 496 = 6$.

Compare the standard deviation: Since the numbers in each set increase by an increment of 1, the standard deviation will be the same.

5. <u>C</u>

The standard deviation is lower when the numbers in a data set are closer to the mean. 72 is closest to the mean.

Note that although 40 is the lowest answer choice, this does not imply that it will decrease the standard deviation the most. In fact, it is farthest from the mean, hence, will increase the standard deviation the greatest when compared with other answer choices.

Category 67 – Compare Mean, Median, Standard Deviation, and Range

1. <u>C</u>

Adding a constant number to each number will not change the range. The mean and the median will change.

2. <u>B</u>

Original data set is 0, 0, 0.2, 0.2, 1.1, 2.5, 4.8.

Data set after replacing 4.8 with 0.8 is 0, 0, 0.2, 0.2, 0.8 1.1, 2.5.

Since 4.8 is the greatest number in the original set, replacing it with 0.8 will decrease the mean, the range and standard deviation. The median $= 0.2$ remains same.

3. <u>C</u>

In the original data set of 21 plants, 14-inch plant is the shortest plant. After the 4-inch plant is added, it is the shortest plant. Hence, the range must increase by $14 - 4 = 10$.

Since the height of all the plants is not given, it is unknown if the mean and the median will increase by 10.

4. <u>C</u>

Mean of Group A:

$$\frac{15 + 18 + 24 + 26 + 32 + 35}{6} = \frac{150}{6} = 25$$

Mean of Group B:

$$\frac{23 + 23 + 25 + 26 + 26 + 27}{6} = \frac{150}{6} = 25$$

Median of Group A: Average of 24 and 26 $= 25$.

Median of Group B: Average of 25 and 26 $= 25.5$.

This eliminates answer choices A, B, and D.

Note that the ranges are different. Group A $= 35 - 15 = 20$. Group B $= 27 - 23 = 4$.

5. <u>C</u>

10% increase will result in higher mean and median.

6. <u>C</u>

Range: Removing the numbers between 50 and 125 will decrease the range by 75, and decrease the standard deviation.

Median: The question is asking for certainty using the phrase "must change". Since the actual numbers of a data set and their frequency is unknown in a box plot, it cannot be determined with certainty how many numbers will be removed and how median will be affected.

7. <u>D</u>

Mean: After adding 5 games, the mean will change.

Median: In 20 games, the 10^{th} and 11^{th} game contains the median score. Median $= 3$.

In 25 games, the 13^{th} game contains the median score. Median $= 3$. Hence, the median does not change.

Range: In 20 games, range $= 5 - 1 = 4$. In 25 games, range $= 6 - 1 = 5$. Hence, the range will change. This eliminates answer choices A, B, and C.

8. <u>D</u>

Median: It is in the 11^{th} data point/dot. Count the dots from left to right till the 11^{th} dot is reached.

At 9 am, the 11^{th} dot is for number of notebooks $= 3$.

At 10 am, The 11^{th} dot is for number of notebooks $= 4$.

Hence, the medians are different.

Standard deviation: Changing the frequency of the data changes the standard deviation.

Category 68 – Interpretation of Sample Data in Studies and Surveys

1. <u>B</u>

The correct sample should be a random sample of teachers from all district schools.

2. <u>C</u>

95% confidence level indicates that the team conducting the study is 95% confident that the results reflect the actual (true) average tusk size of the entire male population of African elephants. The average size was concluded to be from 5.8 to 6.3 feet.

3. <u>D</u>

An associated margin of 15 minutes indicates that the students probably studied from $80 - 15 = 65$ to $80 + 15 = 95$ minutes.

4. <u>C</u>

The sample population in the scientist's experiment was miniature hybrid tea roses from a certain geographic region. Hence, the largest sample population is all the miniature hybrid tea roses in the geographic region.

Section 12 – Drill

1. <u>C</u>

Since the apple production decreased at a constant rate each year over 8-year period, the correlation is negative and steady. This eliminates answer choices A, B, and D.

2. <u>108</u>

Sort the numbers least to greatest:

$$41, 44, 87, 90, 126, 127, 135, 139$$

Determine the middle number: the two middle numbers are 4th and 5th number. Hence, the median is

$$\frac{90 + 126}{2} = \frac{216}{2} = 108$$

3. <u>A</u>

Determine the mean of a and b: Total numbers $= 4$.

$$\frac{x + 11 + 3x + 5}{4} = \frac{4x + 16}{4} =$$

$$\frac{4(x + 4)}{4} = x + 4$$

4. <u>D</u>

Follow the events from the graph:

Since the flat fee is \$25 per month, the monthly cost will start at 25. This eliminates answer choice C.

Since 20 visits are included, there will be a horizontal line till 20. This eliminates answer choice B.

Since after 20 visits in a month, each additional visit $=$ \$2, the monthly cost will go up. This matches the graph in answer choice D.

5. <u>22</u>

Maximum from the box plot $= 22$.

6. <u>C</u>

The most appropriate sampling method is to collect random water samples from all the 6 lakes in the 200 mile vicinity of the manufacturing plant. This matches answer choice C.

7. <u>B</u>

Evaluate each answer choice:

Answer choice B is incorrect since Ken's heart rate was static for 5 mins between 10 and 15 mins and for 5 mins between 25 and 30 mins. Total $= 5 + 5 = 10$ minutes not 15 minutes. All other choices are true.

8. <u>12</u>

Read the height of each bar starting from week 1:

$$6, 5, 14, 18, 16, 10, 9, 18, 17, 6$$

Order the numbers least to greatest:

$$5, 6, 6, 9, 10, 14, 16, 17, 18, 18$$

Determine the median: Since there are 10 weeks, median is the average of the 5th and 6th week.

$$\frac{10 + 14}{2} = 12$$

9. <u>A</u>

Read the height of each bar:

Company A: $14 + 7 + 10 + 13 + 10 + 4 = 58$

Company B: $10 + 4 + 9 + 11 + 12 + 10 = 56$

Determine the difference: $58 - 56 = 2$

10. <u>A</u>

Determine the probability:

There are 9 multiples of 11 between 1 and 100: 11, 22, 33, 44, 55, 66, 77, 88, 99. Four of them are even numbers.

$$\text{probability} = \frac{4}{100} = \frac{1}{25}$$

11. <u>B</u>

Determine the total circumference of all the babies:

$(33 \times 2) + (34 \times 5) + (35 \times 6) + (36 \times 8) +$

$(37 \times 6) + (38 \times 5) =$

$66 + 170 + 210 + 288 + 222 + 190 = 1{,}146$

Determine the mean:

$$\frac{1{,}146}{32} = 35.81$$

12. 80

Determine the sum of numbers:

Since the mean of 7 numbers $= 92$, their sum $=$ $7 \times 92 = 644$.

Since the mean of 6 numbers after removing the lowest number $= 94$, their sum $= 6 \times 94 = 564$.

Hence, the lowest number $= 644 - 564 = 80$.

13. C

Order the numbers least to greatest:

$$3, x, y, z, 121$$

Since x, y, and z are in the middle of the lowest number 3 and highest number 121, the range will not change for any value given to x, y, and z. However, the mean and median will vary depending on the values given to x, y, and z.

14. 48

Determine the participants in Team A:

$$\text{probability} = \frac{\text{all participants in Team A}}{150} = 0.24 \quad \rightarrow$$

$$\text{all participants in Team A} = 0.24 \times 150 = 36$$

Determine the participants in Team B:

$$\text{probability} = \frac{\text{all participants in Team B}}{150} = 0.44 \quad \rightarrow$$

$$\text{all participants in Team B} = 0.44 \times 150 = 66$$

Determine the participants in Team C:

$$150 - (36 + 66) = 48$$

15. C

Median of data set B will be lower since each number is reduced by 24.

Range will be same since the lowest and highest integer in the data set are being subtracted by the same number.

16 B

Read the median from the box plot:

Bakery 1 $= a$: Approximate median $= 28$.

Bakery 2 $= b$: Median $= 25$.

Hence, $a > b$.

17. A

Determine the slope of line of best fit: Below is slope using the points (70, 10) and (90, 40).

$$\frac{40-10}{90-70} = \frac{30}{20} = \frac{3}{2} = \frac{\text{rise}}{\text{run}} = \frac{\text{cost}}{\text{temperature}}$$

Hence, for every 2°F (run), the predicted cost increase (rise) is \$3.

For every 1°F, the predicted cost increase is $\frac{\$3}{2} = \$1.50$.

18. D

Only the employees in the sales department were asked about their liking. Hence, all employees in the sales department is the largest group, the conclusions can be generalized to. This matches answer choice D.

19. C

Use the process of elimination for a question like this and determine the median first.

Determine the median: Since there are 30 adults, the bar that includes the 15th and 16th adult contains the median.

This corresponds to bar 5 -10. Hence, the median can be any number from 5 to less than 10. This eliminates answer choices B and D.

Determine the lowest mean: The lowest mean will be the mean of the left corner numbers for each bar.

$(0 \times 10) + (5 \times 6) + (10 \times 4) + (15 \times 7) +$

$(20 \times 2) + (25 \times 1) =$

$0 + 30 + 40 + 105 + 40 + 25 = 240$

$$\text{mean} = \frac{240}{30} = 8$$

This eliminates answer choice A that has mean $= 7$.

20. D

Mean: 500 is greater than the integers between 185 and 325. Hence, the mean of data set H that includes 500 in addition to 185 and 325 must be greater than the mean of data set G. This eliminates answer choices A and C.

Median: For 45 integers between 185 and 325, ordered from least to greatest, the median is the 23rd integer. Adding 500 to data set H results in 46 integers. The median of 46 integers is the average of 23rd and 24th integer. Hence, the median of data set H could either be the same or greater than that of data set G. This implies that the median of data set G cannot be greater than that of data set H.

21. 0

The median of 25 numbers in a data set is the 13th number from least to greatest. Do a running count of the frequencies for each set data set till the 13th data value.

Data set M: the 13th number is data value $= 8$.

Data set N: the 13th number is data value $= 8$.

Difference $= 8 - 8 = 0$.

22. C

Total of 11 integers $= 33 \times 11 = 363$.

Total of 9 integers $= 30 \times 9 = 270$.

Hence, $270 + a + b = 363 \quad \rightarrow \quad a + b = 93$.

Evaluate each answer choice: $a + b$ must equal 93.

In answer choice C, $40 + 53 = 93$.

23. 1/7 or 0.143 or .1429

Let total number of in-person responses = x.

Hence, total number of email responses = $\frac{x}{3}$.

Yoga+Pilates in-person: Total percent = $35 + 55 =$ 90%.

$$\text{total freshman} = 0.9x$$

Yoga+Pilates email: Total percent = $25 + 20 = 45\%$.

$$\text{total freshman} = (0.45)\frac{x}{3} = 0.15x$$

$$\text{Probability} = \frac{\text{Yoga + Pilates by email}}{\text{Yoga + Pilates by email + in person}} =$$

$$\frac{0.15x}{0.9x + 0.15x} = \frac{0.15x}{1.05x} = \frac{1}{7} = 0.142857$$

24. D

Determine the y-intercept of line of best fit for data set Q:

Dividing the x-intercept by 20 will not change the y-intercept. It is same as that of data set P. From the graph, the y-intercept can be approximated a little above 25. This eliminates answer choices A and C.

Determine the slope of line of best fit for data set Q:

Read a point from the given graph. Point (300, 150) is on the graph. For data set Q, this point is $(\frac{300}{20}, 150) =$ (15, 150). As mentioned above, y-intercept can be approximated as (0, 25). Use these two points to set up a slope equation.

$$\frac{150 - 25}{15 - 0} = \frac{125}{15} = 8.3$$

This is closest to answer choice D.

Section 13 – Geometry

Category 69 – Area and Angles of a Circle

1. A

Convert degrees to radians:

$$60° \times \frac{\pi}{180} = \frac{\pi}{3}$$

2. B

$\angle C = 90°$. Since 90° is one-fourth of a circle, the area of the sector is

$$\frac{\text{area of the circle}}{4} = \frac{12\pi}{4} = 3\pi$$

3. A

Determine the area of the circle: radius = 2.5.

$$\pi r^2 = 3.14 \times 2.5 \times 2.5 = 19.625 = 19.6$$

4. 4

Determine the radius of Circle B: Area of Circle A= 4π.

Area of Circle B = $4 \times 4\pi = 16\pi$. Hence,

$$\pi r^2 = 16\pi \rightarrow r^2 = 16 \rightarrow r = \pm 4 = 4$$

5. B

Determine the area of the circle: Since the diameter is 12, the radius is 6.

$$\text{area of the circle} = \pi r^2 = 36\pi$$

Determine the area of a sector: All 12 sectors are equal.

$$\text{area of each sector} = \frac{36\pi}{12} = 3\pi$$

6. D

Determine the inscribed angle:

$$\frac{\text{central angle}}{2} = \frac{150°}{2} = 75°$$

Convert to radians:

$$75 \times \frac{\pi}{180} = \frac{5\pi}{12}$$

7. B

Convert radians to degree:

$$\frac{2\pi}{3} \times \frac{180}{\pi} = 120°$$

Set up a proportion:

$$\frac{\text{area of a sector}}{\text{area of the circle}} = \frac{120°}{360°} = \frac{1}{3}$$

8. B

Available sector = $360° - 60° = 300°$.

Determine the area of a sector for 300°: Radius = 100.

$$\text{area of sector} = \frac{300}{360}\pi(100)(100) = \frac{25{,}000}{3}\pi$$

9. D

Determine the area of the circles:

$$\text{area of Circle 1 is } \pi r^2 = \pi 5^2 = 25\pi$$

$$\text{area of Circle 2 is } \pi r^2 = \pi 2^2 = 4\pi$$

Determine the ratio of the areas:

$$\text{Circle 1: Circle 2} = 25\pi : 4\pi = 6.25 : 1 = 6.3 : 1$$

10. C

Determine the area of the circles: Since the circles are congruent, the area of the 3 circles is

$$3 \times \pi r^2 = 3 \times \pi 3^2 = 27\pi = 27 \times 3.14 = 84.78$$

Determine the area of shaded region: Total area = 88.

$$\text{shaded area} = 88 - 84.78 = 3.22$$

Category 70 – Circumference and Arc of a Circle

1. B

Determine the length of $\widehat{ADC}$:

The angle corresponding to $\widehat{ADC} = 360° - 60° = 300°$. This is five times more than the angle x corresponding to $\widehat{ABC}$. Hence,

$$\widehat{ADC} = \widehat{ABC} \times 5 = 3\pi \times 5 = 15\pi$$

2. 75

Use the degrees arc length formula: $s = 5\pi$. $r = 12$.

$$5\pi = 2\pi \times 12 \times \frac{\theta}{360} \rightarrow 5 = \frac{24}{360}\theta \rightarrow$$

$$5 = \frac{1}{15}\theta \rightarrow \theta = 5 \times 15 = 75$$

3. 8

Determine the length of minor arc $\widehat{MN}$:

The inscribed angle in degrees is

$$\frac{\pi}{4} \times \frac{180}{\pi} = 45°$$

Hence, $\widehat{MN} = 45° \times 2 = 90°$. This is one-fourth of the circumference. Hence,

$$\widehat{MN} = \frac{\text{circumference}}{4} = \frac{32}{4} = 8$$

4. A

Determine circumference of the circle: $r = 2$.

$$2\pi r = 2\pi \times 2 = 4\pi$$

Determine the fraction:

$$\frac{\text{minor arc}}{\text{circumference}} = \frac{\frac{2\pi}{5}}{4\pi} = \frac{2\pi}{4\pi \times 5} = \frac{1}{10}$$

5. 3

Use the radians arc length formula: $s = 2\pi$. $\theta = \frac{2\pi}{3}$.

$$2\pi = r \times \frac{2\pi}{3} \rightarrow r = \frac{2\pi \times 3}{2\pi} \rightarrow r = 3$$

6. C

Total length traveled $= 1{,}310\pi$.

$a =$ feet in addition to $1{,}280\pi$ feet. This eliminates answer choices A and B.

Determine the revolutions for $1{,}280\pi$ feet: Radius $= 160$

$$\text{number of revolutions} = \frac{1{,}280\pi}{2\pi r} = \frac{1{,}280\pi}{2\pi(160)} = 4$$

Hence, a is the length in addition to 4 revolutions.

Category 71 – Equation of a Circle

1. B

$(x + 4)^2 \rightarrow x$-coordinate $= -4$.

$(y - 11)^2 \rightarrow y$-coordinate $= 11$.

2. D

Determine the equation:

Since the center is $(-7, 0)$, the corresponding factors are $(x + 7)^2$ and y^2. This eliminates choices A and C.

Since $r = 6$, $r^2 = 36$. This eliminates answer choice B.

3. D

Determine the equation:

Since the center is $(-2, -4)$, the corresponding factors are $(x + 2)^2$ and $(y + 4)^2$. The equation can be written as $(x + 2)^2 + (y + 4)^2 = r^2$. This eliminates answer choices A and C.

Determine the radius: Plug the point $(0.5, -2)$ into the above equation.

$$(0.5 + 2)^2 + (-2 + 4)^2 = r^2 \rightarrow (2.5)^2 + (2)^2 = r^2$$

$$r^2 = 6.25 + 4 = 10.25$$

This eliminates answer choice B.

4. C

Translation of 3 units up will change $(y)^2$ to $(y - 3)^2$.

5. A

Evaluate each answer choice: $r^2 = 36$. Hence, any point inside the circle has $r^2 < 36$ and any point outside the circle has $r^2 > 36$. Plug in each point and check.

Evaluate $(-2, 0)$: $(-2 - 2)^2 + (0 + 3)^2 = 16 + 9 = 25$. This point is inside the circle.

Evaluate $(-5, 1)$: $(-5 - 2)^2 + (1 + 3)^2 = 49 + 16 = 65$. This point is outside the circle.

Evaluate $(3, 3)$: $(3 - 2)^2 + (3 + 3)^2 = 1 + 36 = 37$. This point is outside the circle.

6. A

Determine the coordinates of the center:

$$x = \frac{1 - 1}{2} = 0$$

$$y = \frac{-2 - 6}{2} = \frac{-8}{2} = -4$$

Hence, the factors are x^2 and $(y + 4)^2$. The equation can be written as $x^2 + (y + 4)^2 = r^2$. This eliminates answer choices C and D.

Determine the radius: Plug $(1, -2)$ into the above equation. (Same results with $(-1, -6)$).

$$1^2 + (-2 + 4)^2 = r^2 \rightarrow 1 + (2)^2 = r^2 \rightarrow r^2 = 5$$

This eliminates answer choice B.

7. <u>18</u>

Determine the factors for the complete square:

$$x^2 - x + y^2 + 13y = \frac{77}{2} = 38.5$$

Factor for x: Coefficient of x is -1.

$$\frac{-1}{2} = -0.5$$

The complete square is $(x - 0.5)^2$.

Factor for y: Coefficient of y is 13.

$$\frac{13}{2} = 6.5$$

The complete square is $(y + 6.5)^2$.

Complete the equation:

$$(x - 0.5)^2 + (y + 6.5)^2 = 38.5 + (-\mathbf{0.5})^2 + (\mathbf{6.5})^{\mathbf{2}} \rightarrow$$

$$(x - 0.5)^2 + (y + 6.5)^2 = 38.5 + 0.25 + 42.25 = 81$$

$$r^2 = 9^2 \rightarrow r = \pm 9 = 9$$

Hence, diameter is $9 \times 2 = 18$.

8. <u>4</u>

Move $1528p^2$ to the right-side of the equation and divide the equation by the common multiple 8.

$$x^2 + 14px + y^2 - 8py = 191p^2$$

Convert to standard form equation: (See previous answer on how to determine the factors.)

$$(x + 7p)^2 + (y - 4p)^2 = 191p^2 + (7p)^2 + (-4p)^2$$

Hence, $r^2 = 191p^2 + 49p^2 + 16p^2 = 256p^2$.

Equate: Radius is given as np^2. Hence,

$$(n^2p)^2 = 256p^2 \rightarrow n^4p^2 = 256p^2 \rightarrow$$

$$n^4 = 256 \rightarrow n = \sqrt[4]{256} = \pm 4 \rightarrow n = 4$$

Category 72 – Parallel and Intersecting Lines

1. <u>C</u>

Determine x: Since vertical angles are equal, equate the two angles.

$$3x - 5 = 5x - 85 \rightarrow 2x = 80 \rightarrow x = 40$$

2. <u>25</u>

a and b are same side interior supplementary angles. Hence, $a + b = 180$

Substitute $a = 3x + 6$ and $b = 4x - 1$.

$$3x + 6 + 4x - 1 = 180$$

$$7x = 175 \rightarrow x = 25$$

3. <u>98, 100, or 102</u>

$$x + y + y = 180 \rightarrow x + 2y = 180$$

Solve for x:

Since $38 < y < 42$, the possible values of y are 39, 40, 41. Plugging any of these in the above equation will give a possible value of x. See below for all possible values.

$$x + (2 \times 39) = 180 \rightarrow x + 78 = 180 \rightarrow x = 102$$

$$x + (2 \times 40) = 180 \rightarrow x + 80 = 180 \rightarrow x = 100$$

$$x + (2 \times 41) = 180 \rightarrow x + 82 = 180 \rightarrow x = 98$$

4. <u>B</u>

Evaluate each answer choice:

$a + b = 180°$: Since a and b are same side supplementary exterior angle, their sum is 180°. This answer choice is correct.

$a + c = 180°$: Since a and c are angles of a point on a straight line, their sum is 180°. This answer choice is correct.

$b + c = 180°$: b and c are congruent alternate exterior angles. They can only add up to 180 if each of them is 90°. Question does not give this information. Hence, it cannot be concluded that this answer choice must be true.

5. <u>A</u>

Evaluate each answer choice:

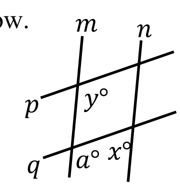
Answer choice A: $k = y + z$.

k and $y + z$ are corresponding angles formed by intersecting line c. This answer choice is correct.

Answer choices B, C, and D are incorrect.

6. <u>65</u>

See the figure below.



y and a are corresponding angles. Hence,

$$y = a = x + 50$$

a and x are same side interior supplementary angles. Hence, $a + x = 180$.

Substitute $x + 50$ for a.

$$a + x = 180 \rightarrow x + 50 + x = 180 \rightarrow$$

$$2x = 180 - 50 = 130 \rightarrow x = 65$$

7. D

Evaluate each answer choice for the types of angles:

$a + y = 120°$: Since y and b are vertical angles, $y = b$. Hence, $a + y = a + b = 120°$. This answer choice is correct.

$b + z = 120°$: Since a and z are vertical angles, $a = z$. Hence, $a + b = z + b = b + z = 120°$. This answer choice is correct.

$c + x = 120°$: Since a, b, and c are angles on a straight line, $a + b + c = 180°$.

$$120° + c = 180° \rightarrow c = 60°$$

Since c and x are vertical angles, $c = x = 60°$. Hence, $c + x = 60° + 60° = 120°$. This answer choice is correct.

8. 9

Set up segment proportion and solve:

$AC = 10$. $CE = 6$. $AE = AC + CE = 10 + 6 = 16$.

$BD = 15$. Let $DF = x$. $BF = BD + DF = 15 + x$.

The length of DF can be calculated by setting up the following proportion.

$$\frac{AE}{CE} = \frac{BF}{DF} \rightarrow \frac{16}{6} = \frac{15 + x}{x}$$

$$16x = 6(15 + x) \rightarrow 16x = 90 + 6x \rightarrow$$

$$10x = 90 \rightarrow x = 9$$

9. B

Set up segment proportion and solve:

$AE = 8.1$. $BD = 4$. $DF = 5$. $BF = BD + DF = 4 + 5 = 9$.

The length of CE can be calculated by setting up the following proportion.

$$\frac{AE}{CE} = \frac{BF}{DF} \rightarrow \frac{8.1}{CE} = \frac{9}{5}$$

$$9 \times CE = 5 \times 8.1 \rightarrow 9 \times CE = 40.5 \rightarrow CE = 4.5$$

This falls in answer choice B.

Category 73 – Polygons

1. B

The sum of interior angles of a quadrilateral $= 360°$.

Solve for x:

$$x + x + 2x + 60 = 360 \rightarrow 4x + 60 = 360$$

$$4x = 360 - 60 = 300 \rightarrow x = 75$$

2. C

Determine n: Each exterior angle $= 30°$.

$$30 = \frac{360}{n}$$

$$30n = 360 \rightarrow n = 12$$

Determine the average degree measure of interior angle:

$$\frac{180(12 - 2)}{12} = \frac{180 \times 10}{12} = 150$$

3. B

Determine the average measure of exterior angle: $n = 10$.

$$\frac{360}{n} = \frac{360}{10} = 36$$

4. 7

Determine the number of sides: Sum of interior angles $= 900°$.

$$180(n - 2) = 900 \rightarrow 180n - 360 = 900$$

$$180n = 900 + 360 \rightarrow 180n = 1{,}260 \rightarrow n = 7$$

5. 40

Determine the perimeter: Each side of regular polygon is identical.

$$\text{perimeter} = AB \times 8 = 5 \times 8 = 40$$

6. 75

Determine the sum of the interior angles: $n = 6$.

$$180(n - 2) = 180(6 - 2) = 180 \times 4 = 720$$

Determine x: Add all the angles and equate to 720.

$$85 + 145 + (2x - 10) + 60 + 150 + (x + 65) = 720$$

$$495 + 3x = 720 \rightarrow 3x = 225 \rightarrow x = 75$$

Category 74 – Angles, Sides, and Area of a Triangle

1. C

$$x + y + z = 180 \rightarrow x = 180 - y - z$$

2. 120

Determine $3x$: Since $3x$ is the exterior angle of triangle XYZ, it is the sum of the two non-adjacent angles.

$$3x = x + 80 \rightarrow 2x = 80 \rightarrow x = 40$$

Hence, $3x = 3 \times 40 = 120$.

3. A

Since angle $M = 91°$, the sum of angles N and O is $180° - 91° = 89°$. Hence, it is not possible for angle N or O to be 89 or greater. This eliminates answer choices B, C, and D. Note that if angle O is 89°, then angle N is 0°. This is not possible.

4. A

Determine the measure of angle A:

Since $AB = AC$, $\angle B = \angle C = 50°$. Hence,

$$\angle A + 50 + 50 = 180 \rightarrow \angle A = 180 - 100 = 80$$

Convert to radians:

$$80 \times \frac{\pi}{180} = \frac{4\pi}{9}$$

5. 8 or 9

Determine the possible lengths of c: $a = 3$. $b = 7$.

c must be less than $3 + 7 = 10$. It is given that $c > 7$.

Hence, $10 > c > 7$.

The possible integer values of c are 8 or 9.

6. D

Determine the measure of angles in triangle ABC:

Since $AB = BC$, $\angle A = \angle C = 40°$. Hence,

$$\angle 40 + 40 + \angle ABC = 180$$

$$\angle ABC = 180 - 80 = 100$$

Determine the measure of angles in triangle BDE:

Since the angles at point B add to 180,

$$\angle DBE = 180 - 100 = 80$$

Since $BE = BD$, $\angle BED = \angle BDE = \angle x$. Hence,

$$\angle x + \angle x + 80° = 180$$

$$2x = 180 - 80 = 100 \rightarrow x = 50$$

7. D

Determine each side:

$$\frac{54}{3} = 18$$

Determine the area:

$$\frac{\sqrt{3}}{4}(18)(18) = 81\sqrt{3}$$

8. B

Determine the angles with same degree measure:

In an isosceles triangle two of the angles have the same degree measure.

$\angle RST = 96°$ cannot be the measure of the two same angles because $96° \times 2 > 180°$. Hence, the sum of two same measure angles SRT and RTS is $180° - 96° = 84°$.

$$\angle SRT = \frac{84}{2} = 42°$$

Convert to radians:

$$42° \times \frac{\pi}{180} = \frac{7}{30}\pi$$

9. B

Determine the possible lengths of c: $a = 2$. $b = 5$.

The three combinations for lengths are

$a + b > c \rightarrow 2 + 5 > c \rightarrow 7 > c$.

$a + c > b \rightarrow 2 + c > 5 \rightarrow c > 5 - 2 \rightarrow c > 3$.

$b + c > a \rightarrow 5 + c > 2 \rightarrow c > 2 - 5 \rightarrow c > -3$.

Hence, $7 > c > 3$.

This eliminates Option I since 2 and 3 are not possible values of c and Option III since 8 is not a possible value of c.

10. B

Determine the side length of equilateral triangle ABC:

Since the area of each congruent triangle = $\sqrt{3}$, the area of ABC is $9 \times \sqrt{3} = 9\sqrt{3}$. Hence,

$$\frac{\sqrt{3}}{4}a^2 = 9\sqrt{3} \rightarrow a^2 = 9 \times 4 = 36 = 6^2 \rightarrow$$

$$a = \pm 6 = 6$$

Hence, each side of triangle $ABC = 6$

Determine the perimeter of equilateral triangle ABC:

$$\text{side } \times 3 = 6 \times 3 = 18$$

11. B

area of shaded region = area of triangle PQR − area of triangle MNR

Determine the area of equilateral triangle PQR: Side = 4.

$$\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}4^2 = 4\sqrt{3}$$

Determine the area of equilateral triangle MNR:

Since MR and NR are the midpoint of PR and QR, respectively, and $MN = 2$, triangle MNR is an equilateral triangle with each side = 2.

$$\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4}2^2 = \sqrt{3}$$

Determine the area of shaded region:

$$4\sqrt{3} - \sqrt{3} = 3\sqrt{3}$$

Category 75 – Right Triangles Side Ratios and Theorems

1. <u>D</u>

Use the Pythagorean theorem:

$$c^2 = a^2 + b^2 \rightarrow c = \pm\sqrt{a^2+b^2} \rightarrow c = \sqrt{a^2+b^2}$$

2. <u>96</u>

Determine the height AB: base $= BC = 16$.

$BC{:}AC = 16{:}20$ is the ratio of the Pythagorean triple $(3{:}4{:}5)4 = 12{:}16{:}20$. Hence,

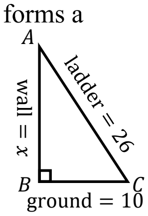
$$AB{:}BC{:}AC = 12{:}16{:}20 \rightarrow AB = 12$$

Determine the area:

$$\frac{1}{2} \times bh = \frac{1}{2} \times (16)(12) = 96$$

3. <u>24</u>

Determine the height: Ladder against the wall forms a right triangle. See below.



$26 =$ hypotenuse $= AC$. $10 =$ one leg $= BC$.
Height of wall $= x =$ second leg $= AB$.

$BC{:}AC = 10{:}26$ is the ratio of the Pythagorean triple $(5{:}12{:}13)2 = 10{:}24{:}26$.

Hence, $AB{:}BC{:}AC = 10{:}24{:}26 \rightarrow AB = 24$

4. <u>C</u>

Determine x: Use the Pythagorean theorem.

$$AB^2 + BC^2 = AC^2 \rightarrow x^2 + (2x)^2 = 10^2$$

$$x^2 + 4x^2 = 100 \rightarrow 5x^2 = 100 \rightarrow x^2 = 20$$

$$x = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

Determine the perimeter of triangle ABC:

$$x + 2x + 10 = 2\sqrt{5} + \left(2 \times 2\sqrt{5}\right) + 10 = 6\sqrt{5} + 10$$

5. <u>4</u>

Since $AB = BC$ and angle $ABC = 90°$, triangle ABC is a right isosceles 45°-45°-90° triangle with side ratio as

$$AB{:}BC{:}AC = a{:}a{:}a\sqrt{2} = a{:}a{:}4\sqrt{2}$$

$$a\sqrt{2} = 4\sqrt{2} \rightarrow a = 4 = AB$$

6. <u>8</u>

Set up area: Area $= 56$. Leg 1 $= x$. Leg 2 $= x + 6$.

$$56 = \frac{1}{2}(x)(x+6)$$

Determine x: Set up and solve quadratic equation.

$$(2)(56) = x^2 + 6x \rightarrow x^2 + 6x - 112 = 0 \rightarrow$$

$$(x-8)(x+14) \rightarrow x = 8 \text{ and} -14$$

Since length cannot be negative, $x = 8$.

7. <u>4</u>

Determine the length of AC: Since AD bisects BC, $\angle A = 30° + 30° = 60°$. Hence, $\angle C = 60°$ and triangle ABC is an equilateral triangle divided into two equal 30°-60°-90° triangles.

In triangle ADC, the side ratio are as follows.

$$CD{:}AD{:}AC = a{:}a\sqrt{3}{:}2a = a{:}2\sqrt{3}{:}2a$$

$$a\sqrt{3} = 2\sqrt{3} \rightarrow a = 2$$

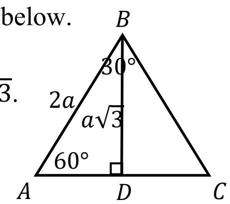
$$AC = 2a = 2 \times 2 = 4$$

8. <u>73</u>

Determine each side: Perimeter $= 438$.

$$\frac{438}{3} = 146$$

Determine the length of the perpendicular line segment: AD divides ABC into two 30°-60°-90° triangles. See the figure and the ratio components below.



side $= 2a = 146$.

Hence, $a = 73$ and $a\sqrt{3} = 73\sqrt{3}$.

$$b\sqrt{3} = 73\sqrt{3} \rightarrow b = 73$$

9. <u>6</u>

Determine $\overline{NR}$:

$$NR^2 = MR \times RQ = 9 \times 4 = 36 \rightarrow$$

$$NR = \sqrt{36} = \pm 6 = 6$$

10. <u>25</u>

Equate with the side ratio of 45°-45°-90° traingle:

The side ratio is $a{:}a{:}a\sqrt{2}$.

Perimeter can be written as $2a + a\sqrt{2}$. Hence,

$$2a + a\sqrt{2} = k + 12.5\sqrt{2} \rightarrow a = 12.5$$

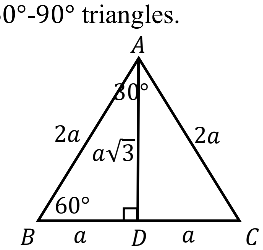
Hence, $k = 2a = 2(12.5) = 25$.

11. <u>30</u>

Determine each side: Perimeter $= 60\sqrt{3}$.

$$\frac{60\sqrt{3}}{3} = 20\sqrt{3}$$

Determine the height: The height AD in the figure below divides the triangle into two 30°-60°-90° triangles. See the ratio components below.



$2a = 20\sqrt{3}$.

Hence, $a = 10\sqrt{3}$ and

$AD = a\sqrt{3} = \left(10\sqrt{3}\right)\left(\sqrt{3}\right) = 30$

12. <u>C</u>

Determine the area of triangle XYZ:

The largest length $= \sqrt{51}$ must be the hypotenuse. Hence, the base and height must be the other two lengths.

$$\frac{1}{2}bh = \frac{1}{2} \times \sqrt{3} \times 4\sqrt{3} = 6$$

Category 76 – Similar Triangles

1. <u>A</u>

Angle A corresponds to angle M. Hence, they have the same degree measure.

2. <u>B</u>

Determine the corresponding sides: AB corresponds to EF. BC corresponds to DE. AC corresponds to DF.

$$\frac{AB}{AC} = \frac{EF}{DF}$$

3. <u>46</u>

Determine $\angle A$: $\angle B = 70°$. $\angle C = 32°$.

$$\angle A + 70 + 32 = 180 \rightarrow \angle A = 180 - 102 = 78$$

Since the triangles are similar, the corresponding vertices will have the same degree measure.

$\angle X = \angle A = 78$ and $\angle Z = \angle C = 32$.

Determine $\angle X - \angle Z$: $78 - 32 = 46$

4. <u>20</u>

Since $\angle B = \angle D = 90°$ and $\angle C = \angle E$ (corresponding angles), the two triangles are similar triangles.

Set up proportion and solve:

In right triangle ADE, $AD: AE = 6: 10 = 2(3: 5)$ is the ratio of Pythagorean triple $2(3: 4: 5)$. Hence,

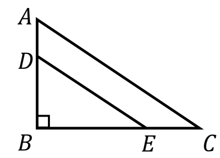
$$AD: DE: AE = 6: DE: 10 = 6: 8: 10 \rightarrow DE = 8$$

The length of BC can be determined by setting up the following proportion.

$$\frac{AB}{AD} = \frac{BC}{DE} \rightarrow \frac{6+9}{6} = \frac{BC}{8} \rightarrow \frac{15}{6} = \frac{BC}{8} \rightarrow$$

$$6 \times BC = 8 \times 15 \rightarrow 6 \times BC = 120 \rightarrow BC = 20$$

5. <u>6</u>



Joining points D and E creates a right triangle DBE. See the figure above.

Since $\angle C = \angle E$ and $\angle A = \angle D$ (corresponding angles), the two triangles are similar triangles

Set up proportion and solve: Let $CE = x$.

Hence, $BE = 2x$, and $BC = BE + CE = x + 2x = 3x$.

Set up a proportion as shown below.

$$\frac{BC}{BE} = \frac{AC}{DE} \rightarrow \frac{3x}{2x} = \frac{9}{DE} \rightarrow \frac{3}{2} = \frac{9}{DE} \rightarrow$$

$$3 \times DE = 9 \times 2 \rightarrow 3 \times DE = 18 \rightarrow DE = 6$$

6. <u>10</u>

Triangles MNP and MQN are similar.

In triangle MNP, $\overline{MP}$ can be determined using the Pythagorean theorem.

$$MP^2 = MN^2 + NP^2 = \left(10\sqrt{3}\right)^2 + \left(10\sqrt{6}\right)^2 =$$

$$300 + 600 = 900 \rightarrow MP = \sqrt{900} = 30$$

$\overline{MQ}$ can be determined by setting up the following proportion, using the corresponding sides. (Numerator is for ΔMNP. Denominator is for ΔMQN.)

$$\frac{MN}{MP} = \frac{MQ}{MN} \rightarrow MQ = \frac{(MN)(MN)}{MP} =$$

$$\frac{(10\sqrt{3})(10\sqrt{3})}{30} = 10$$

Category 77 – Squares and Cubes

1. <u>B</u>

Determine the length of each side: Area = 100.

$$s^2 = 100 = 10^2 \rightarrow s = \pm 10 \rightarrow s = 10$$

2. <u>1/4 or 0.25</u>

Determine the surface area of one face of the cube:

$$6s^2 = 6\left(\frac{1}{2}\right)^2$$

Hence, surface area of one face is

$$\left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$$

3. <u>80</u>

Determine the length of each side: Volume = 64.

$$s^3 = 64 = 4^3 \rightarrow s = 4$$

Determine the difference between the surface area and area of one face of the cube:

Since each face is a square, the difference is

$$6s^2 - s^2 = 5s^2 = 5 \times 4^2 = 5 \times 16 = 80$$

4. <u>150</u>

Determine the length of each side: Diagonal = $5\sqrt{3}$.

$$s\sqrt{3} = 5\sqrt{3} \rightarrow s = 5$$

Determine the surface area of the cube:

$$6s^2 = 6 \times 5^2 = 6 \times 25 = 150$$

5. <u>125</u>

Determine the length of each side: Surface area = 150.

$$6s^2 = 150 \rightarrow s^2 = 25 \rightarrow s = \pm 5 = 5$$

Determine the volume of the cube:

$$s^3 = 5^3 = 125$$

6. <u>C</u>

Determine the volume of both the cubes:

For cube with $s = 5$, volume $= s^3 = 5^3 = 125$.

For cube with $s = 2$, volume $= s^3 = 2^3 = 8$.

Difference: $125 - 8 = 117$.

7. A

Determine the length of each side: Area $= \frac{b^2}{4}$.

$$s^2 = \frac{b^2}{4} \rightarrow s^2 = \frac{b^2}{2^2} \rightarrow s^2 = \left(\frac{b}{2}\right)^2 \rightarrow s = \pm\frac{b}{2} = \frac{b}{2}$$

Determine the perimeter of the square:

$$4s = 4 \times \frac{b}{2} = 2b$$

8. C

Determine side length of square B:

Perimeter of square B is $x + 76$. Hence, each side of square B is

$$\frac{x + 76}{4} = 0.25x + 19$$

Determine the area of square B (function f):

$$f(x) = s^2 = (0.25x + 19)^2$$

Category 78 – Rectangles and Right Rectangular Prisms

1. B

Determine the perimeter of the rectangle:

$$2(l + w) = 2(14 + 17) = 2(31) = 62$$

2. B

Determine the surface area of the right rectangular prism:

$$2(lw + lh + wh) = 2\big((7 \times 5) + (7 \times 2) + (5 \times 2)\big)$$
$$2(35 + 14 + 10) = 2 \times 59 = 118$$

3. D

Determine the volume (function f):

Width $= w$. Height $= 3w$. Length $= 2 \times 3w = 6w$.

$$f(w) = lwh = 6w \times w \times 3w = 18w^3$$

4. A

Determine the width of the rectangle: Area $= 147$.

Let width $= x$. Hence, length $= 3x$.

$$x \times 3x = 147 \rightarrow 3x^2 = 147 \rightarrow x^2 = 49 \rightarrow$$
$$x = \pm 7 = 7$$

Determine the difference:

$$3x - x = 2x = 2 \times 7 = 14$$

5. B

$f(l)$ is the perimeter based on the length of l.

Hence, $f(45) = 918$ is the perimeter when $l = 45$ inches.

6. C

Determine the surface area of each identical right rectangular prism: $w = 1.2x$. $l = 2.5x$. $h = 8$.

$$2(lw + lh + wh) =$$
$$2\big((2.5x \times 1.2x) + (2.5x \times 8) + (1.2x \times 8)\big) =$$
$$2(3x^2 + 20x + 9.6x) = 6x^2 + 59.2x$$

Hence, surface area of 2 identical prisms is

$$2(6x^2 + 59.2x) = 12x^2 + 118.4x$$

Since one lw of each prism is lost in gluing, the lost surface area of both prisms is $2lw = 2 \times 3x^2 = 6x^2$.

Hence, surface area of glued prisms is

$$12x^2 + 118.4x - 6x^2 = 6x^2 + 118.4x$$

Category 79 – Trapezoids and Parallelograms

1. 320

Determine the area of the parallelogram: $b = 16$. $h = 20$.

$$16 \times 20 = 320$$

2. 10

Determine the height of the trapezoid: Area $= 110$.

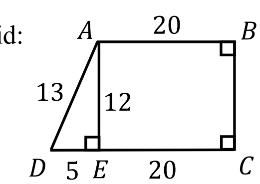
The two bases are 6.5 and 15.5.

$$110 = \frac{1}{2}(6.5 + 15.5)h \rightarrow 110 = \frac{1}{2}(22)h \rightarrow$$
$$110 = 11h \rightarrow h = 10$$

3. 270

Determine the height of the trapezoid:

See the figure on the right.



A perpendicular line drawn from point A is the height of the trapezoid. It divides the trapezoid into right triangle AED and rectangle $ABCE$. Hence, $AB = CE = 20$ and $DE = 25 - 20 = 5$.

$DE: AD = 5: 13$ is the ratio of Pythagorean triple $5: 12: 13$. Hence,

$$DE: AE: AD = 5: AE: 13 = 5: 12: 13 \rightarrow AE = 12$$

Determine the area of the trapezoid:

$$\frac{1}{2}(20 + 25)12 = 45 \times 6 = 270$$

4. <u>55</u>

Determine the length of the sides of the parallelogram:

$DE: AE = 10: 24 = 2(5: 12)$ is the ratio of Pythagorean triple $2(5: 12: 13)$. Hence,

$$DE: AE: AD = 10: 24: AD = 10: 24: 26 \rightarrow AD = 26$$

Since perimeter = 162,

$$AD + BC + AB + DC = 162$$

Since the opposite sides of a parallelogram are equal in length, the perimeter can be written as

$$2(AD) + 2(AB) = 162 \rightarrow 2(26) + 2(AB) = 162 \rightarrow$$
$$52 + 2(AB) = 162 \rightarrow 2(AB) = 110 \rightarrow AB = 55$$

5. <u>12</u>

Determine the length of the side of the parallelogram:
Since $\angle O = 60°$ and $\angle Q = 90°$, $\angle M$ is 30° and triangle MOQ is a 30°-60°-90° triangle. Hence,

$$OQ: MQ: MO = a: a\sqrt{3}: 2a = a: 3\sqrt{3}: 2a$$
$$a\sqrt{3} = 3\sqrt{3} \rightarrow a = 3 = OQ$$
$$OP = OQ + QP = 3 + 9 = 12$$

Since the opposite sides of a parallelogram are equal in length, $OP = MN = 12$.

Category 80 – Volume of Cylinders, Spheres, Cones, Pyramids, Prisms

1. <u>B</u>

Determine the volume of the cylinder: $h = 5.5$.
Diameter = 6. Hence, radius = 3.

$$\pi r^2 h = \pi(3^2)(5.5) = (9)(5.5)\pi = 49.5\pi$$

2. <u>C</u>

Determine the height of the cylindrical pipe: $V = 1{,}728\pi$. $r = 6$.

$$1{,}728\pi = \pi(6)^2 h \rightarrow 1{,}728 = 36h \rightarrow h = 48$$

3. <u>113</u>

Determine the volume of the sphere: = amount of air
Diameter = 6. Hence, radius = 3.

$$\frac{4}{3}\pi \times 3^3 = 36\pi = 36 \times 3.14 = 113.04$$

4. <u>4</u>

Determine the radius of the sphere: $V = \frac{32}{3}\pi$.

$$\frac{32}{3}\pi = \frac{4}{3}\pi r^3 \rightarrow$$
$$32 = 4r^3 \rightarrow r^3 = 8 = 2^3 \rightarrow r = 2$$

Determine diameter: $2 \times 2 = 4$

5. <u>A</u>

Determine the area of the equilateral triangular base:
Each side of the equilateral triangle = 2.

$$\frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 2 \times 2 = \sqrt{3}$$

Determine the volume of the prism:

$$\sqrt{3} \times 3\sqrt{3} = 3 \times 3 = 9$$

6. <u>9</u>

Determine the radius of the cylindrical cone: $V = 27\pi h$.

$$27\pi h = \frac{1}{3}\pi r^2 h \rightarrow 27 = \frac{1}{3}r^2 \rightarrow$$
$$r^2 = 27 \times 3 = 81 = 9^2 \rightarrow r = \pm 9 = 9$$

7. <u>42</u>

Determine the volume of the cylinder: $h = 6$.
Area occupied = area of base = $\pi r^2 = 7$.

$$V = \pi r^2 h = 7 \times 6 = 42$$

8. <u>6</u>

Determine the height of the cylinder: $V = 54\pi$. $h = 2r$.

$$54\pi = \pi r^2 \times 2r \rightarrow 54\pi = 2\pi r^3 \rightarrow$$
$$r^3 = 27 = 3^3 \rightarrow r = \pm 3 = 3$$
$$h = 2 \times 3 = 6$$

9. <u>14</u>

Determine the side of the pyramid square base:

each side of square = $\frac{\text{perimeter}}{4} = \frac{12}{4} = 3$

Determine the height of the pyramid: $V = 42$. $l = w = 3$.

$$42 = \frac{1}{3}lwh \rightarrow 42 = \frac{1}{3} \times 3 \times 3 \times h \rightarrow h = 14$$

10. <u>B</u>

Determine the volume of the cylindrical cone:
It is given that $h = 3r$.

$$V = \frac{1}{3}\pi r^2 3r \rightarrow V = \pi r^3$$

11. <u>16</u>

Determine the area of the pyramid base: $V = 32$. $h = 6$.

$$32 = \frac{1}{3}\text{ area of base} \times 6 \rightarrow \text{ area of base} = \frac{32}{2} = 16$$

12. <u>B</u>

Determine the volume of both containers:
Let $x = \pi r^2 h$.
Hence, $y = \pi(2r)^2 2h = \pi 4r^2 2h = 8\pi r^2 h$.

$$\frac{x}{y} = \frac{\pi r^2 h}{8\pi r^2 h} = \frac{1}{8}$$

13. C

Determine the radius of the original circular cone: Volume = 6π. $h = 2$.

$$6\pi = \frac{1}{3}\pi r^2 \times 2 \rightarrow r^2 = \frac{6 \times 3}{2} = 9 \rightarrow r = \pm 3 = 3$$

Determine the volume of circular cone with double r and h: $r = 3 \times 2 = 6$. $h = 2 \times 2 = 4$.

$$\frac{1}{3}\pi \times 6 \times 6 \times 4 = 48\pi$$

Change in volume:

$$\frac{48\pi}{6\pi} = 8 \text{ times increase}$$

14. D

Determine the radius of the cylindrical cone: $h = 12$.

Since the volume should be $\leq 64\pi$, determine the radius for the maximum volume = 64.

$$64\pi = \frac{1}{3}\pi r^2 h \rightarrow 64 = \frac{1}{3}r^2 \times 12 \rightarrow$$

$$64 = 4r^2 \rightarrow r^2 = \frac{64}{4} = 16 = 4^2 \rightarrow r = \pm 4 = 4$$

Hence, for volume $\leq 64\pi$, r can not be greater than 4.

Category 81 – Mass, Volume, and Density relationship

1. B

Determine the mass: Density = 562. Volume = $lwh = 0.75 \times 0.65 \times 0.95 = 0.463$ approximately.

mass = density × volume = $562 \times 0.463 = 260.27625$. This is close to answer choice B.

2. A

Determine the volume: Density = 1.05. Mass = 0.84.

$$\text{volume} = \frac{\text{mass}}{\text{density}} = \frac{0.84}{1.05} = 0.8$$

Determine the height of the pyramid: $lw = 3$. $V = 0.8$.

$$V = \frac{1}{3}lwh \rightarrow 0.8 = \frac{1}{3} \times 3 \times h \rightarrow h = 0.8$$

Category 82 – Combined Geometric Figures

1. D

Since $AE = BE$, $\angle ABE = \angle BAE = \frac{90}{2} = 45$.

Since $BC || DE$ (opposite parallel sides of a trapezoid) and $\angle BEA = 90°$, $\angle CBE = 90°$.

$$\angle ABE + \angle CBE = 45 + 90 = 135$$

2. 50

Since $\widehat{AB} = \widehat{AC}$, the opposite angles are congruent.

$$\angle B = \angle C$$

Since arc $BXC = 160°$, inscribed $\angle A = 80°$. Hence,

$$\angle B + \angle C = 180 - 80 = 100$$

Since $\angle B = \angle C$, each angle = $\frac{100}{2} = 50$.

3. 72

The length l of the rectangle is the diameter of the circle, and the width w is the radius of the circle.

Determine the radius of the circle:

$$\frac{\pi r^2}{2} = 18\pi \rightarrow r^2 = 2 \times 18 = 36 \rightarrow r = \pm 6 \rightarrow$$

$$r = 6 = w$$

$$\text{diameter} = 2 \times r = 2 \times 6 = 12 = l$$

Determine the area of the rectangle:

$$lw = 12 \times 6 = 72$$

4. 120

Since $\angle B = 90°$, triangle ABC is a right triangle with base = BC and height = AB.

Since $r = 13$, AC = diameter = $2 \times 13 = 26$.

$BC{:}AC = 10{:}26 = 2(5{:}13)$ is the ratio of Pythagorean triple $2(5{:}12{:}13)$. Hence,

$$BC{:}AB{:}AC = 10{:}AB{:}26 = 10{:}24{:}26 \rightarrow AB = 24$$

Determine the area of triangle ABC:

$$\frac{1}{2}bh = \frac{1}{2}BC \times AB = \frac{1}{2} \times 10 \times 24 = 120$$

5. 40

$$\angle A = \frac{5}{9}\pi = \frac{5}{9}\pi \times \frac{180}{\pi} = 100°$$

Since AB and AC are the radii, $AB = AC$ and $\angle B = \angle C$. Hence,

$$100 + \angle B + \angle C = 180 \rightarrow$$

$$\angle B + \angle C = 180 - 100 = 80$$

Since $\angle B = \angle C$, each angle = $\frac{80}{2} = 40$.

6. 60

Since lines a and b are tangent to the circle, $\angle CBD$ and $\angle CAD$ are 90°.

The sum of the angles of a quadrilateral = 360°. Hence,

$$\angle CBD + \angle CAD + x + \angle ADB = 360$$

$$90 + 90 + 120 + \angle ADB = 360$$

$$300 + \angle ADB = 360 \rightarrow \angle ADB = 360 - 300 = 60$$

Since $\angle ADB$ and $\angle y$ are vertical angles, $\angle y = \angle ADB = 60$.

7. <u>32</u>

BC is a side of the equilateral triangle and the square. Since the area of the equilateral triangle is given, the length of BC can be determined.

Determine the sides of the equilateral triangle ABC:

$$\frac{\sqrt{3}}{4}a^2 = 16\sqrt{3} \rightarrow a^2 = 16 \times 4 = 64 = 8^2 \rightarrow$$
$$a = \pm 8 = 8 = BC$$

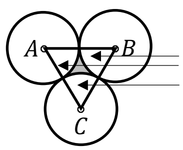
Determine the perimeter of square $BCDE$:

$$4s = 4 \times BC = 4 \times 8 = 32$$

8. <u>B</u>

Since each circle is congruent, the centers A, B, and C form an equilateral triangle with each side $= 2 \times$ radius $= 2 \times 6 = 12$ and $\angle A = \angle B = \angle C = 60°$.

area of the shaded region = area of triangle $ABC -$ the area of 3 circle sectors. See the figure below.



Determine the area of equilateral triangle ABC:

$$\text{area} = \frac{\sqrt{3}}{4} \times 12 \times 12 = \sqrt{3} \times 36 = 36\sqrt{3}$$

Determine the area of 3 sectors:

$$\text{area of 1 sector} = \frac{\theta}{360} \times \pi r^2 = \frac{60}{360} \times \pi \times 6 \times 6 = 6\pi$$
$$\text{area of 3 sectors} = 3 \times 6\pi = 18\pi$$

Determine the area of the shaded region:

$$36\sqrt{3} - 18\pi = 18(2\sqrt{3} - \pi)$$

9. <u>D</u>

Use the process of elimination:

Since neither the square nor the inscribed rectangle have a square term in the sides, $3a^2 + 4$ cannot be the perimeter of the square or the rectangle. This eliminates answer choices A and B.

Evaluate answer choice C: The area of the square and rectangle combined is

$$4a^2 + (a+2)(a-2) = 4a^2 + a^2 - 4 = 5a^2 - 4$$

This eliminates answer choice C.

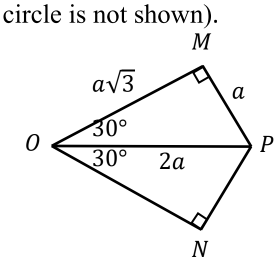
10. <u>C</u>

Determine the radius of the circle:

Line segments OM and ON are the radii of the circle.

Line segments OM, ON, MP, and NP form quadrilateral $OMPN$.

A line segment from O to P divides $OMPN$ into two 30°-60°-90° triangles. See the figure below for the ratio of sides (the circle is not shown).



Since $MP = a = 26$, OM = radius $= a\sqrt{3} = 26\sqrt{3}$.

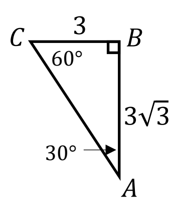
Determine the area of the circle:

$$\pi r^2 \rightarrow \pi(26\sqrt{3})(26\sqrt{3}) = 2{,}028\pi$$

Category 83 – Geometric Figures in the xy-plane

1. <u>D</u>

See the figure below.



Determine the length between points: AB is a vertical line and BC is a horizontal line. Hence, $\angle B$ is 90°.

Determine $\overline{AB}$: $3\sqrt{3} - 0 = 3\sqrt{3}$.

Determine $\overline{BC}$: $1 - (-2) = 3$.

$BC : AB = 3 : 3\sqrt{3}$ is the ratio of the sides of a 30°-60°-90° right triangle. Hence,

$BC : AB : AC = 3 : 3\sqrt{3} : AC = a : a\sqrt{3} : 2a = 30°:60°:90°$

Determine $\angle A$:

$\angle A$ is opposite to $BC = a$. Hence, $\angle A = 30°$.

$$30 \times \frac{\pi}{180} = \frac{\pi}{6}$$

2. <u>4</u>

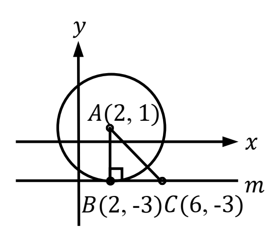
Determine the length between points using the distance formula: Radius is the distance between the points.

$$\sqrt{(1-0)^2 + (\sqrt{3}-0)^2} \rightarrow \sqrt{1 + (\sqrt{3})^2} \rightarrow \sqrt{4} = 2$$

Diameter $= 2 \times 2 = 4$.

3. <u>B</u>

See the figure below.



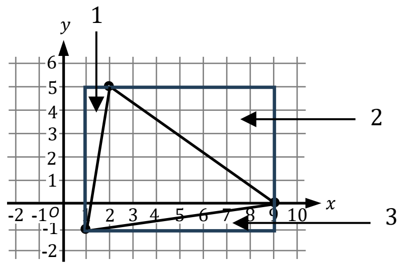
Determine the length between points: AB is a vertical line and BC is a horizontal line. Hence, $\angle B$ is 90°.

Determine $\overline{AB}$: $1 - (-3) = 1 + 3 = 4$.

Determine $\overline{BC}$: $6 - (2) = 4$.

Since $AB = BC$, triangle ABC is an isosceles right triangle. Hence, $\angle A = \angle C = 45°$.

4. A



Since it is not easy to determine the height of the triangle connected by the given points, enclose the triangle in a rectangle. Area of the triangle formed by 3 points = area of rectangle − 3 corner triangles. See figure above. The 3 corner triangles are marked with numbers.

Determine the base and height of the rectangle and each corner triangle: Read the distance between the points.

Rectangle: Base = 8. Height = 6. Area = $8 \times 6 = 48$

Triangle 1: Base = 1. Height = 6. Area = $\frac{1}{2} \times 1 \times 6 = 3$

Triangle 2: Base = 7. Height = 5. Area = $\frac{1}{2} \times 7 \times 5 = 17.5$

Triangle 3: Base = 8. Height = 1. Area = $\frac{1}{2} \times 8 \times 1 = 4$

Determine the area of the triangle enclosed within the points:

$$48 - (3 + 17.5 + 4) = 48 - 24.5 = 23.5$$

Category 84 – Geometric Figures and Percent

1. B

Determine the current distance:

$$AB + BC = 5 + 12 = 17$$

Determine the distance after bypass: The length of bypass $= AC$ is not given.

$AB: BC = 5: 12$ is the ratio of Pythagorean triple $5: 12: 13$. Hence,

$$AB: BC: AC = 5: 12: 13 \rightarrow AC = 13$$

Determine the percent change:

Old distance = 17. New distance = 13.

$$\frac{13 - 17}{17} \times 100 = \frac{-4}{17} \times 100 = -23.5\% = 23.5\%$$

2. A

Determine the area of the rectangle:

Let length = l and width = w.

$$\text{area} = lw$$

Determine the area of the modified rectangle:

Length increased by $10\% = 1.1l$.

Width decreased by $20\% = 0.8w$.

$$\text{area} = 1.1l \times 0.8w = 0.88lw$$

Difference: $0.88lw - lw = -0.12lw = 12\%$ less

3. D

Determine the area before side increase:

Let side = x. Hence, area = x^2.

Determine the area after 40% side increase:

$$40\% \text{ increase} = 1 + 0.4 = 1.4$$

Hence, increased side = $1.4x$.

$$\text{area} = (1.4x)^2 = 1.96x^2$$

Determine the percent change:

$$\frac{1.96x^2 - x^2}{x^2} \times 100 = \frac{0.96x^2}{x^2} \times 100 = 96\%$$

4. C

Determine the area before radius increase: Let radius $= r$.

$$\text{area} = \pi r^2 = \pi 4^2 = 16\pi$$

Determine the area after 50% radius increase:

Increased radius = $1 + 0.5 = 1.5r = 1.5 \times 4 = 6$.

$$\pi r^2 = \pi 6^2 = 36\pi$$

Determine the percent change:

$$\frac{36\pi - 16\pi}{16\pi} \times 100 = \frac{20\pi}{16\pi} \times 100 = 125\%$$

5. C

Determine x:

$$30\% \text{ increase} = 1 + 0.3 = 1.3$$

Hence, the area after 30% increase of $x = 1.3x$.

Each side of the square with above area = 8. Hence,

$1.3x = 8^2 = 64 \rightarrow x = 49.23$ = area in initial design.

Determine the side before increase:

$$\text{area} = s^2 = 49.23 = \text{approximately } 7^2$$

Since 49.23 is greater than $7^2 = 49$, the side cannot be less than 7. This eliminates answer choices A and B.

Answer choice D can be eliminated since the area of $s = 8$ is $s^2 = 8^2 = 64$.

Category 85 – Ratio of Linear Lengths in Similar Geometric Figures

1. B

Determine the ratio of lengths of the corresponding sides:

$$\text{trapezoid } K\text{:trapezoid } L = 2:5$$

Determine the ratio of areas:

$$\text{trapezoid } K\text{:trapezoid } L = 2^2:5^2 = 4:25$$

Determine area of K is what percent of L:

$$\frac{4}{25} \times 100 = 16$$

2. A

Determine the cube ratio of the volumes:

$$64:1000 = 4^3:10^3$$

Hence, ratio of the corresponding heights $= 4:10 = 2:5$.

Determine the proportion:

$$2:5 = h:4 \rightarrow h = 1.6$$

Section 13 – Drill

1. D

Determine the sum of the degree measure of interior angles: $n = 5$.

$$180(n-2) = 180(5-2) = 180 \times 3 = 540$$

Determine x: The unknown angle at point P must be determined first. Since the angles at point P add to 180°, the unknown angle $= 180 - 50 = 130$.

$$\angle x + 120 + 60 + 130 + 90 = 540$$

$$\angle x + 400 = 540 \rightarrow \angle x = 540 - 400 = 140$$

2. C

Determine the relationship:

The given triangle is a right-angle triangle. Hence, use the Pythagorean Theorem.

$$a^2 + 15^2 = 22^2$$

3. 52

Determine the surface area of the rectangle:

$$2(l + w) = 2(10.5 + 15.5) = 2(26) = 52$$

4. B

Determine k:

The three angles where lines q and m intersect are: right angle, angle k, and an unknown angle. Hence,

$$\text{angle } k + \text{unknown angle} = 90°$$

The unknown angle is the exterior alternate angle of angle $a = 48°$. Hence,

$$k + 48 = 90 \rightarrow k = 90 - 48 = 42$$

5. A

Area $= x$. $b = AC$. $h = BD = 10$.

$$x = \frac{1}{2} \times AC \times 10 \rightarrow x = 5AC \rightarrow AC = \frac{x}{5}$$

6. 48

Since $AC||DE$, $\angle A = \angle D$. Hence,

$$\angle A = \angle D = 52$$

$$\angle A + \angle B + \angle C = 180 \rightarrow 52 + \angle B + 80 = 180$$

$$\angle B = 180 - 132 = 48$$

7. D

Determine the volume of cylinder Y: $h = 10$.
$r = 2 \times 3 = 6$.

$$V = \pi r^2 h = \pi \times 6 \times 6 \times 10 = 360\pi$$

8. 6

Determine the radius of the circle, using area of sector formula: $\theta = 120°$. Area of sector $= 12\pi$.

$$12\pi = \frac{120}{360} \times \pi r^2 \rightarrow 12 = \frac{1}{3} \times r^2 \rightarrow r^2 = 36 = 6^2$$

$$r = \pm 6 \rightarrow r = 6$$

9. C

Determine $\angle M$: $\angle M$ is the central angle for $\widehat{NP}$. Hence, they have the same degree measure. Since $\angle N$ is a right angle, triangle MNQ is a right triangle.

$MN:NQ = 1:\sqrt{3}$ is the ratio of the sides of a 30°-60°-90° right triangle. Hence,

$$MN:NQ:MQ = 1:\sqrt{3}:MQ = a:a\sqrt{3}:2a = 30°:60°:90°$$

Since angle $\angle M$ is opposite to NQ $(a\sqrt{3})$, $\angle M = 60°$. Hence, $\widehat{NP} = 60°$.

Determine the ratio:

$$\widehat{NP}\text{: circumferenece} = 60°:360° = 1:6$$

10. 9

Determine the width of the rectangle: Length $= l$. Width $= l + 5$. Area $= 36$.

$$36 = l \times (l + 5) \rightarrow 36 = l^2 + 5l$$

Form a quadratic equation and factor.

$$l^2 + 5l - 36 = 0$$

$$(l + 9)(l - 4) = 0 \rightarrow l = -9 \text{ or } l = 4$$

Length is greater than 0. Hence, $l = 4$

$$\text{width} = l + 5 = 4 + 5 = 9$$

11. <u>5.3</u>

Since $\angle B = \angle E$ and $\angle C$ is shared, triangles ABC and CED are similar triangles.

$$BE = BC - CE$$

In triangle CED, $DE: CD = 3: 5$ is the ratio of Pythagorean triple $3: 4: 5$. Hence,

$$DE: CE: CD = 3: CE: 5 = 3: 4: 5 \rightarrow CE = 4$$

Hence, $BE = BC - CE = BC - 4$.

The length of BC can be determined by setting up the following proportion.

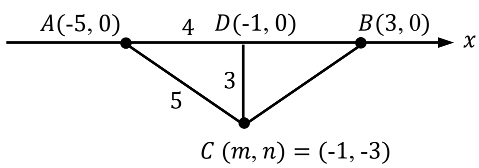
$$\frac{BC}{CE} = \frac{AB}{DE} \rightarrow \frac{BC}{4} = \frac{7}{3}$$

$$3 \times BC = 4 \times 7 \rightarrow 3 \times BC = 28 \rightarrow BC = 9.33$$

$$BE = BC - CE = 9.33 - 4 = 5.33 = 5.3$$

12. <u>B</u>

See partial figure below.



Determine m: A perpendicular bisector from the center C to the x-axis is the midpoint of the two points shown on the x-axis. The x-coordinate of this midpoint, D, is the x-coordinate of the center of the circle.

$$\text{midpoint } D = m = \frac{-5 + 3}{2} = \frac{-2}{2} = -1$$

Determine n: The length of CD will give the position of n on the y-axis. See below using triangle ACD.

$AC = 5$ since it is the radius.

$AD = -1 - (-5) = 4$

$AD: AC = 4: 5$ is the ratio of Pythagorean triple $3: 4: 5$.

$$CD: AD: AC = CD: 4: 5 = 3: 4: 5 \rightarrow CD = 3$$

n is 3 units below 0 on the y-axis. Hence, $n = -3$.

13. <u>C</u>

Move $28by$ to the left-side of the equation.

$$4x^2 + 20ax + 4y^2 - 28by = 20c$$

Divide the equation by 4.

$$x^2 + 5ax + y^2 - 7by = 5c$$

Determine the factor for x:

$$\frac{5a}{2}$$

Determine the factor for y:

$$\frac{-7b}{2}$$

Hence, the left-side of the standard form equation is

$$\left(x + \frac{5a}{2}\right)^2 + \left(y - \frac{7b}{2}\right)^2$$

The coordinates of the center are

$$\left(-\frac{5a}{2}, \frac{7b}{2}\right)$$

14. <u>B</u>

The line segment (radius) from the center of the circle to the point $(3, 11)$ is perpendicular to the tangent line p. Hence, the slope of line p is the negative reciprocal of the slope of the line segment (radius).

Determine the slope of line segment p: Center $= (0, 14)$.

Set up a slope equation using the points $(3, 11)$ and $(0, 14)$ to determine the slope of the radius from the center to the point $(3, 11)$ on the circle.

$$\text{slope} = \frac{14 - 11}{0 - 3} = \frac{3}{-3} = -1$$

Hence, the slope of line p is 1 (negative reciprocal of -1).

Evaluate each answer choice: Set up a slope equation for each answer choice using $(3, 11)$ and the point in the answer choice. The correct answer choice will have slope $= 1$.

Answer choice A:

$$\text{slope} = \frac{-14 - 11}{-25 - 3} = \frac{-25}{-28}$$

Answer choice A can be eliminated.

Answer choice B:

$$\text{slope} = \frac{-7 - 11}{-15 - 3} = \frac{-18}{-18} = 1$$

Answer choice B is correct. No need to check for the remaining answer choices.

15. <u>30</u>

Determine the volume of the right rectangular prism:

$lw = 5.\ hw = 6.\ lh = 30.$

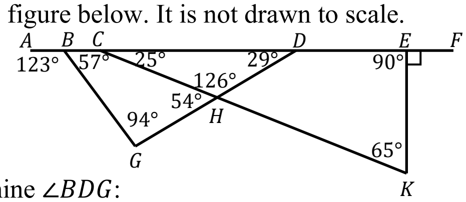
Multiplying the given numbers will give the square of the volume.

$$lw \times hw \times lh = 5 \times 6 \times 30 \rightarrow llwwhh = 900$$

$$(lwh)^2 = (30)^2 \rightarrow lwh = 30$$

16. <u>94</u>

See the figure below. It is not drawn to scale.



Determine $\angle BDG$:

Step 1: $\angle CEK + \angle CKE + \angle ECK = 180 \rightarrow$

$$90 + 65 + \angle ECK = 180 \rightarrow$$

$$\angle ECK = 180 - 90 - 65 = 25$$

Step 2: $\angle CHD = 180 - \angle CHG$ (angles on a line) $\rightarrow$

$$180 - 54 = 126$$

Step 3: $\angle CDH = 180 - \angle CHD - \angle ECK =$

$$180 - 126 - 25 = 29$$

Step 4: $\angle BDG = \angle CDH = 29$ (they are same angles)

Determine $\angle DBG$:

$$\angle DBG = 180 - \angle ABG = 180 - 123 = 57$$

Determine $\angle BGD$:

$$\angle BGD = 180 - \angle DBG - \angle BDG =$$

$$180 - 57 - 29 = 94$$

17. 2

Determine hypotenuse: Use the Pythagorean theorem.

$$\text{hypotenuse}^2 = 12^2 + 18^2 = 144 + 324 = 468 \rightarrow$$

$$\text{hypotenuse} = \sqrt{468} = \sqrt{4 \times 117} = 2\sqrt{117}$$

Equate with the given value.

$$a\sqrt{117} = 2\sqrt{117} \rightarrow a = 2$$

18. 35

Determine hypotenuse:

1: 2: 3 is the ratio of 30°-60°-90° triangle. Hence, the side ratio of the triangle is $a: a\sqrt{3}: 2a$.

Hence, the longest side = $2a$ = hypotenuse.

$$\text{perimeter in terms of } a = a + a\sqrt{3} + 2a = 3a + a\sqrt{3}$$

Equate with the given perimeter.

$$3a + a\sqrt{3} = 52.5 + 17.5\sqrt{3}$$

Hence, $a = 17.5$ and $2a = 2(17.5) = 35$.

19. 605

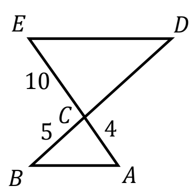
Determine the length of each side:

volume = $s^3 = 1{,}331 = 11^3 \rightarrow s = 11$

Determine the surface area of 5 sides: Box has 6 sides. Since one side is glued, remaining sides = 5.

$$5s^2 = 5(11)^2 = 5(121) = 605$$

20. 17.5



Points D and E connect with triangle ABC to form triangle CDE. See the figure above.

Since $AB||DE$, $\angle A = \angle E$ and $\angle B = \angle D$. Hence, triangles ABC and CDE are similar triangles.

$$BD = BC + CD = 5 + CD$$

The length of CD can be determined by setting up the following proportion.

$$\frac{AC}{CE} = \frac{BC}{CD} \rightarrow \frac{4}{10} = \frac{5}{CD}$$

$$4 \times CD = 5 \times 10 \rightarrow 4 \times CD = 50 \rightarrow CD = 12.5$$

$$BD = 5 + 12.5 = 17.5$$

21. 2

Determine the radius:

Area of equilateral triangle = $4\sqrt{3}$. Hence,

$$\frac{\sqrt{3}}{4}a^2 = 4\sqrt{3} \rightarrow a^2 = 4 \times 4 = 4^2 \rightarrow$$

$$a = \pm 4 = 4$$

Each side of the triangle is comprised of two radii. Hence,

$$\text{radius} = \frac{4}{2} = 2$$

22. 54

Determine each side of the cube: Volume = 64.

$$s^3 = 64 = 4^3 \rightarrow s = 4$$

Determine the surface area of reduced cube:

25% reduction = $1 - 0.25 = 0.75$.

$$\text{reduced side} = 4 \times 0.75 = 3$$

$$6s^2 = 6 \times 3^2 = 6 \times 9 = 54$$

23. 54

Determine the area of $ABCD$: Let length = l and width = w. Hence, area = lw.

$ABCD$ is one-fourth of $MNOP$. Hence, the length and width of $MNOP$ are 4 times of $ABCD$.

$$\text{area of } MNOP = 4l \times 4w = 16lw = 864 \rightarrow$$

$$lw = \frac{864}{16} = 54$$

Area of $ABCD = lw = 54$.

24. 36

Determine the diameter of the circle:

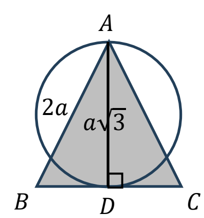
$$\pi r^2 = 27\pi \rightarrow r^2 = 27 \rightarrow r = \pm\sqrt{27} \rightarrow$$

$$r = \pm\sqrt{9 \times 3} \rightarrow r = \pm 3\sqrt{3} = 3\sqrt{3}$$

$$\text{diameter} = 3\sqrt{3} \times 2 = 6\sqrt{3}$$

Determine the length of the sides of triangle ABC:

Since all 3 angles are the same, ABC is an equilateral triangle. Since AD is perpendicular to BC, it divides the triangle into two equal 30°-60°-90° triangles with side ratio $a: a\sqrt{3}: 2a$. See the figure below.



Hence,

$$\text{diameter} = a\sqrt{3} = 6\sqrt{3}$$

$$a = 6$$

$$\text{length of each side of } ABC = 2a = 2 \times 6 = 12$$

Determine the perimeter of triangle ABC:

$$\text{perimeter} = 12 \times 3 = 36$$

25. 929.5

Determine AC: $BD = \sqrt{240}$ and $CD = (60)(AD)$.

$$BD^2 = (AD)(CD) = (AD)(60)(AD) = (60)(AD)^2 \rightarrow$$

$$\left(\sqrt{240}\right)^2 = (60)(AD)^2 \rightarrow 240 = (60)(AD)^2 \rightarrow$$

$$(AD)^2 = \frac{240}{60} = 4 = 2^2 \rightarrow AD = \pm 2 = 2$$

Hence, $CD = (60)(AD) = 60 \times 2 = 120$

Determine area of triangle BDC:

$$\frac{1}{2} \times CD \times BD = \frac{1}{2} \times 120 \times \sqrt{240} = 929.5160$$

Section 14 – Trigonometry

Category 86 – Right Triangles and Trigonometry

1. <u>B</u>

$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{DE}{EF} = \frac{6}{12} = \frac{1}{2}$$

2. <u>0.8 or 4/5</u>

$$\cos(90 - b) = \sin b = \frac{4}{5}$$

3. <u>D</u>

Answer choice D is incorrect, since sin 60° = cos (90° − 30°) = cos 60° is not true.

4. <u>D</u>

Determine the ratio of sides:

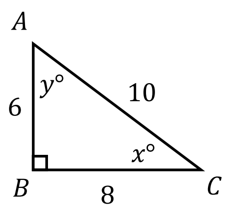
$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{DE}{EF} = 0.75 = \frac{75}{100} = \frac{3}{4}$$

Determine the proportion: It is given that $EF = 12$. This is three times of 4 (from above). Hence, DE is three times of 3.

$$DE = 3 \times 3 = 9$$

5. <u>3/4, 6/8, or 0.75</u>

Determine the ratio of sides: See the figure below.



$$\sin y = \frac{\text{opposite}}{\text{hypothenuse}} = \frac{BC}{AC} = 0.8 = \frac{8}{10}$$

$BC{:}\,AC = 8{:}\,10 = 2(4{:}\,5)$ is the ratio of Pythagorean triple $2(3{:}\,4{:}\,5)$. Hence,

$$AB{:}\,BC{:}\,AC = 6{:}\,8{:}\,10 \rightarrow AB = 6$$

$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{BC} = \frac{6}{8} = \frac{3}{4} = 0.75$$

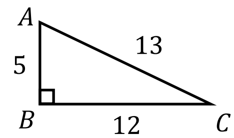
6. <u>15</u>

$$(k + 20) + (3k + 10) = 90$$

$$4k + 30 = 90 \rightarrow 4k = 90 - 30 = 60 \rightarrow k = 15$$

7. <u>C</u>

Determine the ratio of sides: See the figure below.



Either acute angle could be used for the given value of tan. Angle C is used below.

$$\tan C = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{BC} = \frac{5}{12}$$

$AB{:}\,BC = 5{:}\,12$ is the ratio of Pythagorean triple $5{:}\,12{:}\,13$. Hence,

$$AB{:}\,BC{:}\,AC = 5{:}\,12{:}\,13 \rightarrow AC = 13$$

The other acute angle is A.

$$\cos A = \frac{\text{adjacent}}{\text{hypothenuse}} = \frac{5}{13}$$

8. <u>B</u>

The tangent of the two acute angles are inverse of each other.

9. <u>A</u>

Determine the ratio of sides:

$$\tan m = \frac{\text{opposite}}{\text{adjacent}} = \frac{XY}{YZ} = \frac{1}{\sqrt{3}}$$

$XY{:}\,YZ = 1{:}\,\sqrt{3}$ is the ratio of the sides of a 30°-60°-90° right triangle. Hence,

$$XY{:}\,YZ{:}\,XZ = a{:}\,a\sqrt{3}{:}\,2a$$

Since $\angle m$ is opposite XY (smallest side a), $\angle m = 30°$.

10. <u>C</u>

Evaluate each answer choice by replacing sin $x°$ for k: The answer choices must be true for all values of k.

Answer choice A: cos $(x^2)° =$ sin $x°$ is not correct.

Answer choice B: sin $(x^2)° =$ sin $x°$ is not correct.

Answer choice C: cos $(90° - x°) =$ sin $x°$ is correct.

Answer choice D: tan $(90° - x°) =$ sin $x°$ is not correct.

11. <u>B</u>

Evaluate each answer choice: $\angle A = \angle X$, $\angle B = \angle Y$, and $\angle C = \angle Z$.

Option I: sin $A =$ cos C and cos $C =$ cos Z. Hence, sin $A =$ cos Z .This is correct.

Option II: Since $AB \neq BC$, $\angle A \neq \angle C$. Hence, $\angle X \neq \angle Z$ and tan $\angle X \neq$ tan $\angle Z$. Since tan $\angle Z =$ tan $\angle C$, tan $\angle C$ cannot be equal to tan $\angle X$. This eliminates option II.

Option III: sin $(90° - X) =$ cos $A \rightarrow$ sin $(90° - A) =$ cos A. This is correct.

12. D

Determine the ratio of sides:

$$\tan x = \frac{\text{opposite}}{\text{adjacent}} = \frac{DE}{EF} = \frac{1}{1}$$

$DE: EF = 1: 1$ is the ratio of the sides of a 45°-45°-90° right triangle. Hence,

$$DE: EF: DF = a: a: a\sqrt{2} = 1: 1: \sqrt{2}$$

Determine the proportion:

It is given $DE = 5$. Hence, $a = 5$.

$$DE: EF: DF = a: a: a\sqrt{2} = 5: 5: 5\sqrt{2} \rightarrow DF = 5\sqrt{2}$$

13. C

Determine $\tan R$:

$$\tan R = \tan N \text{ (similar triangles)}$$

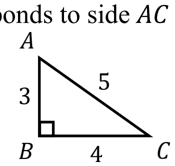
$MN: LM: LN = 36: 48: 60 = 12(3: 4: 5)$ is the ratio of Pythagorean triple 3: 4: 5. The largest side LN is the hypotenuse and angle $M = 90°$. Hence,

$$\tan R = \tan N = \frac{\text{opposite}}{\text{adjacent}} = \frac{LM}{MN} = \frac{4}{3}$$

14. 20

Determine the ratio of sides: See the figure below.

Side XZ corresponds to side AC (based on the vertices).



$$\tan C = \frac{\text{opposite}}{\text{adjacent}} = \frac{AB}{BC} = \frac{3}{4}$$

$AB: BC = 3: 4$ is the ratio of Pythagorean triple 3: 4: 5.

$$AB: BC: AC = 3: 4: 5 \rightarrow AC = 5$$

Determine the proportion: It is given that $BC = 8$. This is twice of 4 (from above). Hence, AC is twice of 5.

$$AC = 2 \times 5 = 10$$

Determine XZ: It is given that each side of triangle ABC is half of the corresponding side of triangle XYZ. Hence,

$$XZ = 2AC = 2 \times 10 = 20$$

15. C

Evaluate each answer choice: sin47° = cos43°.

Adding the two fractions cannot result in squared terms given in answer choices A and B. This eliminates answer choices A and B.

Neither sin47° nor cos43° can result in the values of cos47° or sin43°. This eliminates answer choice D.

In the correct answer choice C, the numerator of the given fraction $\frac{\cos 43°}{\sin 47°}$ is same as sin47°, and the denominator is same as cos43°. See below.

$$\frac{\sin 47°}{\cos 43°} + \frac{\cos 43°}{\sin 47°} = \frac{\sin 47°}{\cos 43°} + \frac{\sin 47°}{\cos 43°} = 2\left(\frac{\sin 47°}{\cos 43°}\right)$$

16. A

Set up proportion: Triangles DGH and DEF are similar. Hence, the corresponding sides are in proportion.

$$\tan F = \tan H = \frac{DG}{GH} = 1.6 = \frac{16}{10} = \frac{8}{5}$$

17. 25

Determine the ratio of sides:

$$\angle \sin C = \sin F \text{ (similar triangles)}$$

$$\sin C = \sin F = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{DE}{DF} = \frac{6}{10}$$

$DE: DF = 6: 10 = 2(3: 5)$ is the ratio of Pythagorean triple $2(3: 4: 5)$. Hence,

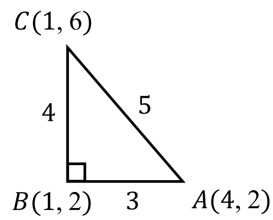
$$DE: EF: DF = 6: 8: 10 \rightarrow EF = 8$$

Determine the proportion: It is given that $EF = 20$. This is 2.5 times of 8 (from above). Hence, DF is 2.5 times 10

$$DF = 2.5 \times 10 = 25$$

18. 4/5 or 0.8

Determine the length between the points: See the figure below.



AB is a horizontal line and BC is a vertical line.

Determine AB: $4 - 1 = 3$.

Determine BC: $6 - 2 = 4$.

$AB: BC = 3: 4$ is the ratio of Pythagorean triple 3: 4: 5.

$$AB: BC: AC = 3: 4: 5 \rightarrow AC = 5$$

$$\sin A = \frac{\text{opposite}}{\text{hypothenuse}} = \frac{BC}{AC} = \frac{4}{5}$$

Category 87 – Unit Circle and Trigonometry

1. D

Determine the coordinates on the unit circle:

The ($\cos 45°$, $\sin 45°$) coordinates in a unit circle are $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$$\tan 45 = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 1$$

2. C

Convert radians to degrees:

$$\frac{\pi}{3} \times \frac{180}{\pi} = 60°$$

Determine the coordinates on the unit circle:

$\theta = 60°$ is in quadrant I. All coordinates are positive in this quadrant. This eliminates answer choices A and B.

1 is not a coordinate of $\theta = 60°$ in a unit circle. This eliminates answer choice D.

3. A

Determine the coordinates on the unit circle:

Angle θ is in quadrant II. Hence, $\cos\theta$ is negative and $\sin\theta$ is positive.

Determine $\tan\theta$:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\sqrt{3}}{2} \div \frac{-1}{2} = \frac{\sqrt{3}}{2} \times -\frac{2}{1} = -\sqrt{3}$$

4. B

Determine the coordinates on the unit circle:

Since point S is in quadrant IV, angle θ is also in quadrant IV. Hence, the x-coordinate $= \cos\theta$ is positive, and the y-coordinate $= \sin\theta$ is negative.

Since $\cos\theta = \frac{\sqrt{2}}{2}$, $\sin\theta$ must be $\frac{-\sqrt{2}}{2}$.

5. C

Convert radians to degrees:

$$\frac{111\pi}{18} \times \frac{180}{\pi} = 1110°$$

Determine the number of $360°$ rotations and coterminal angle:

Divide by $360°$ to determine the number of full $360°$ rotations.

$$\frac{1110}{360} = 3.083$$

Hence, there 3 rotations. This is $360° \times 3 = 1080°$.

Coterminal angle $= 1110° - 1080° = 30°$.

Determine $\tan 30°$:

$\cos 30° = \frac{\sqrt{3}}{2}$ and $\sin 30° = \frac{1}{2}$.

$$\tan\theta = \frac{\sin 30°}{\cos 30°} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} =$$

$$\frac{1}{2} \div \frac{\sqrt{3}}{2} = \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$